

NUMERICAL SOLUTION FOR OPEN CHANNEL FLOW WITH SUBMERGED FLEXIBLE VEGETATION

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ABSTRACT

The purpose of this study is to explore the suitability of numerical models in estimation of velocity and flow resistance (Manning n) in open channels with totally submerged flexible vegetation. A three dimensional (3D) numerical model based on arbitrary Lagrangian-Eulerian (ALE) approach has been employed to simulate the effects of various characteristics of selected flexible vegetations to the velocity distribution and flow resistance. The modeling involved simultaneous solution of Navier Stokes equation for open channel flow, stress-strain relationship for the vegetation structure and ALE algorithm for the moving vegetation boundaries. The numerical computation has been carried out with an aid of a commercial finite element software package, COMSOL Multiphysics 3.4. The numerical results were validated using experimental data carried out in the laboratory using real vegetations. The accuracy of numerical model compared to experimental results was measured in terms of mean absolute error (MAE). The results show that the numerical model which combined the three applications as mentioned above able to predict the velocity and the flow resistance coefficient in open vegetated channel with reasonable accuracy. The MAE calculated for velocity and Manning n is ± 0.02 .

Keywords: *Vegetated channel, vegetative resistance coefficient, numerical model, ALE.*

INTRODUCTION

Knowledge on flow behavior and resistance and the way they response to changing vegetations characteristics, is essential and has many applications in the field of hydraulic engineering, especially in the design and management of vegetated waterways. Interactions between vegetation and flow are complex because of the diverse ways in which vegetation affects the flow fields. Vegetation may create different flow patterns and impose variable degree of resistance depending on its density, size, flexibility, and its position in the flow field. Due to complex nature of fluid-vegetation interaction, most works in the past have been experimental based [1-5] and only a few have attempted theoretical approach [6-7]. The experimentally derived Manning formula remains the most commonly used open channel flow equation, but the resistance coefficient is difficult to accurately determine as it depends on many interlinked factors [8-9].

Experimental work is usually laborious and time consuming, especially when a wide range of flow and vegetation characteristics are to be considered. The theoretical approach uses less physical resources but however involves complex equations which normally require tedious numerical calculations. In the last few years, with an advancement of computer technology and software, which facilitate the computations of complex equations, the theoretical approach through numerical modeling has gain increasing attention. The numerical models enable tests in a wide range of flow and vegetation characteristics to be carried out in more detail and faster than possible with experiments.

Studies have shown that there is great potential in using numerical model in solving flow in channel with rigid or flexible vegetation [10-12]. Kutija and Hong [10] developed one dimensional model based on conservation of momentum coupled with mixing length theory for the assessment of additional resistance due to flexible vegetation. The deflection of the vegetation is calculated using cantilever beam theory. Fischer-antze et al. [11] carried out three dimensional modeling using SSIIM program developed by Olsen [13] for open channel flow with rigid submerged vegetation. The flow is modeled using Reynolds averaged Navier Stokes equation and the additional resistance provided by the vegetation is modeled by introducing drag force term into the Navier Stokes equation. Later Erduran and Kutija [12] introduced three dimensional numerical model for the simulation of flow through flexible and rigid vegetation in submerged and non submerged conditions. The model was developed by coupling shallow water equation with Navier Stokes equation. Both studies [10, 13] used

experimental data given by Tsujimota & Kitamura [14] for rigid vegetation to verify their models. The models were not verified with flexible vegetation experimental data due to unavailability of suitable data.

In this study a combination of Navier Stokes equation, stress-strain relationship and moving mesh method which based on arbitrary Lagrangian Eulerian (ALE) algorithm have been employed to simulate flow through flexible submerged vegetation in three dimensional. ALE technique has an advantage of simulating large deformation. The models are verified against experimental data using real flexible vegetation.

Lagrangian algorithm is most commonly used in structure mechanics, and in this study it can be used to track the vegetation motion. The computational mesh is attached to the vegetation material and as the vegetation boundaries move and deform the mesh move with it and map out the deflection profile. In a case of large deformation, the mesh will become excessively distorted, the Lagrangian method alone will have difficulty providing a reasonable accurate results.

Lagrangian method is also not suitable to track the motion of fluid as the particles of fluid are not cohesive and travel independently of each other. If the mesh is attached to the fluid particles, the mesh will severely deform and may overlap to each other. So for fluid motion the Eulerian algorithm is usually applied. In Eulerian method, the mesh is fixed in space and the fluid particles motion is observed at fixed points. It does not track the path of any individual particle. The ALE algorithm combines the use of two algorithms and it is very useful in study of fluid-vegetation interaction in which large deformation of vegetation structure and the surrounding fluid is expected. It tracks the vegetation motion to some extent, but when the meshes distort beyond an acceptable level, it adjust the meshes and the Eulerian method will take place.

MATERIALS AND METHODS

This study involved simulation of open channel flow with flexible submerged vegetation using numerical models. The modeling was performed with an aid of commercial finite element software package COMSOL Multiphysics 3.4 (COMSOL). Output from the numerical model is compared to experimental results carried out in a laboratory using real vegetations. Results from experiments provide measurements of the boundary conditions for the numerical models and also quantify velocity and resistance coefficient in terms of magnitude and profile for comparison to the numerical output. Simulation of flexible vegetation requires input of modulus of elasticity of the vegetation. The modulus of elasticity of the vegetation is estimated using Board Drop test as illustrated in Kouwen [15] and simple cantilever test as described in Freeman et al. [16] and the average of the results is used in the numerical model. Flow in vegetated channel is inherently three dimensional, with a complex relationship between the flow behaviour and the vegetation characteristics. In order to take into consideration the full three dimensional aspect of the flow surrounding the vegetation, the approach of solving the full three dimensional form of the models is considered.

Experiments

The experiments were carried out in a laboratory flume of 22 m long, 0.6 m wide and 0.9 m depth with a fixed slope of 0.001. The test area over which the vegetation is planted was 8 m long and 7.5 m from the inlet of the flume. Two types of natural vegetation namely *Lepironia Articulata* (LA) and *Scirpus Grossus* (SG) were tested. The description of the vegetation used in the experiments is described in Table 1. The flow rate into the flume is measured using an ultrasonic flow meter (Model UFX, Dynasonics, USA) and experiments were carried out at a constant flow rate of 0.033 m³/s. The three dimensional velocities in the test area were measured using two acoustic Velocity Doppler velocitimeter (ADV), that are SonTek 16-Mhz MicroADV (Sontek/YSI, USA) and Vectrino 25-Mhz (Nortek AS, Norway).

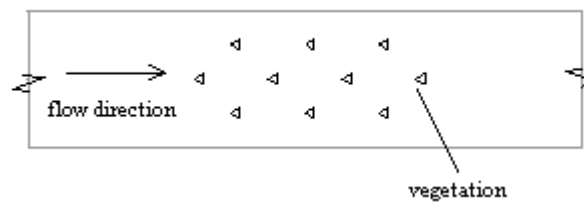
Numerical Solution

Numerical modeling for flow in vegetated channel using COMSOL Multiphysics involved identification of appropriate equations to represent the problem, discretization of the problem into a finite set of elements or meshes, application of the selected equations to these elements, applications of boundary conditions at the edges of the computational meshes, and solution of the resultant equations in an iterative manner.

Table 1: Characteristics of vegetations

Types of vegetation	Characteristics	Vegetation height (m)	Average frontal width (m)	Modulus of elasticity per bunch (N/m ²)	Number of vegetation bunch and distribution*
SG	Long leaf with a triangular shape of 10 -20 mm width, grow in a bunch	0.6	0.1	1.4 x 10 ⁸	10
LA	Leafless, small round flexible stem of 2 mm to 5 mm diameter, grow in a bunch	0.6	0.05	1.7 x 10 ⁷	10

* Vegetation distribution



Plan view of the distribution of the vegetations in the channel

When flow passed through flexible submerged vegetation, the vegetation forced the flow into a narrower path between vegetation stems. The flow imposed viscous drag and pressure force on the vegetation structure and as a result the vegetation structure deformed and bent. Consequently the flow surrounding the vegetation also deformed and produced different magnitude of force. The deformation may be large depending on the extent of the vegetation’s deflection. The vegetation in turn responded to the new force and the process continued in a feedback loop. This is a typical fluid-structure interaction problem and the solution to this problem usually involves coupling the fluid dynamic and structure mechanics models. In this study the fluid flow is modeled by Navier Stokes equations and the vegetation structure is by stress-strain relationship. The dynamic of the deformed geometry and the moving boundaries of the vegetation are handled by moving mesh and ALE methods.

The Navier-stokes equation comprises of momentum equation (eqn 1) and continuity equation (eqn 2), and for steady incompressible flow, the equations are as follows]

$$\rho v \cdot \nabla v = P + \mu \nabla^2 v + F \tag{1}$$

$$\nabla \cdot \rho v = 0 \tag{2}$$

where ρ is the water density, μ the viscosity of the water, v is the velocity in x, y and z direction $\{v_x, v_y, v_z\}$, P is the pressure and F is the external force. The vegetation deformation is solved using stress-strain relationship (eqn 3 and 4) as follows

$$\{\epsilon_g\} = [B]\{\delta\} \tag{3}$$

$$\{\sigma\} = [D]\{\epsilon_g\} \tag{4}$$

Where ϵ_g is the strain, δ is the displacement, σ is the stress, B is the matrices of strain-displacement and D is the matrices of elasticity

The geometry of the model is three dimensional and consists of two domains that are fluid domain and vegetation domain, as shown in Figure 1(a). The fluid domain is 8 m length and 0.6 m width and of the same size and shape as the test reach in the open channel used in the laboratory. The depth of the fluid domain is set at 0.7 m to simulate the depth of flow recorded in the experiment. The vegetation domain is idealized by vertical triangular prisms of the same height, frontal area and modulus of elasticity as the real vegetation tested in the

experiments. The frontal area is the area that obstructs the flow in normal direction and it is measured as the average width of the vegetation multiply with the height of the vegetation. The vegetation is positioned in the model channel geometry according to the density and distribution employed in the laboratory.

For domain discretization, unstructured tetrahedron mesh has been employed (Figure 1 (b)) as this type of mesh can satisfactorily fit to the vegetation domain and the narrow spaces in between vegetation bunch. Relatively fine mesh was distributed in the regions near the vegetation where the large velocity gradient is expected. The finer the mesh the more accurate are the results but solving using fine mesh demand more computation time and higher computer memory, therefore a compromise between accuracy of simulation and computational efficiency is required. The mesh convergence test was conducted to determine suitable mesh size that can give reasonable accurate results with reasonable computation time. From the mesh convergence study, it is found that the increased the number of mesh more than 70,000 with degree of freedom 300,000 did not change much the results of the velocity profile. Therefore the number of mesh employed for the simulation is in the range 70,000-100,000.

For the flow domain, the boundary condition at the inlet is the velocity at the entrance of the test reach measured in experimental works. At the channel bed and sides, the no slip boundary condition is applied, on the assumption that the flow is inviscid. The no slip boundary condition means the velocity flow at the channel boundary in all directions are equal to 0. The slip boundary condition is applied at the channel free surface, which restrict the movement of flow in normal direction but allow the flow in tangential direction.

At the interface of fluid and vegetation, two types of boundary conditions were tested, i.e (1) no slip condition (2) velocity profile inside the vegetation canopy measured in the experiments. The former boundary condition implies no flow in streamwise direction can pass through the vegetation domain while the latter indicates otherwise. The no slip condition at the interface of fluid and vegetation is specifically true if the vegetation stems are closely packed together and the amount of flow that can pass through is insignificant.

The vegetation domain deflected under the imposed fluid force, thus, the boundary condition applied to the surface of vegetation domain is the fluid load per unit area. The vegetation's bottom edge is fixed to the channel bed so its movement is restricted in all directions. All other boundaries are allowed to move freely in any directions depending on the amount of load imposed by surrounding fluid.

The meshes at the fluid-vegetation interface need to be allowed to move following the movement of the vegetation boundaries. Thus the moving mesh boundary condition is applied. The velocity of the fluid is set equal to the velocity of the vegetation motion and the mesh displacement at the fluid-vegetation interface is set equal to the vegetation displacement.

Having set up the mesh and specified all the necessary boundary conditions, the governing equations are solved numerically by using finite element method. The governing equations are nonlinear and in COMSOL the nonlinear equation is solved iteratively by damped Newton Method [17-18]. The method involved discretized the equation into linearised equations which can either be solved by direct or iterative method. For large problem with degree of freedom exceeding 100,000, COMSOL suggest iterative procedure GMRES (Generalised Minimum Residual) with preconditioner Geometric Multigrid [18]. The nonlinear iteration will stop and give the results when the specified convergence criterion has been satisfied. Convergence criterion set the relative tolerance for error. In this study the relative tolerance of 1×10^{-6} has been used in all the simulations. This indicates that the iterations will only stop when the computed relative error is less than the relative tolerance specified.

RESULTS AND DISCUSSION

The experiments and numerical solution for flow in a channel with vegetation of *Scirpus Grossus* and *Lepironia Articulata* have been accomplished. Both numerical and experimental have been run at a steady discharge of $0.033 \text{ m}^3/\text{s}$ and at a channel slope of 0.001. The vegetation shape has been simplified to uniform shape of triangular prism but with the same frontal area, modulus of elasticity and distribution as the real vegetation used in the experiment. The terms SG10-1 and LA10-1 are used in the discussion referring to the simulation for channel with ten *Scirpus Grossus* and ten *Lepironia Articulata*, respectively, and with the boundary condition at the fluid-vegetation interface (1) that is no slip. The terms SG10-2 and LA10-2 refer to the simulation using the boundary condition (2) that is the velocity inside the vegetation canopy obtained experimentally. The output of the simulation and experiments are mainly the variation of velocity distribution and roughness coefficient along

the vegetated area in the channel. Figure 2 and 3 show the results of the simulation for SG10-1 and LA10-1, respectively, in term of velocity distribution along the channel and the displacement of the vegetations.

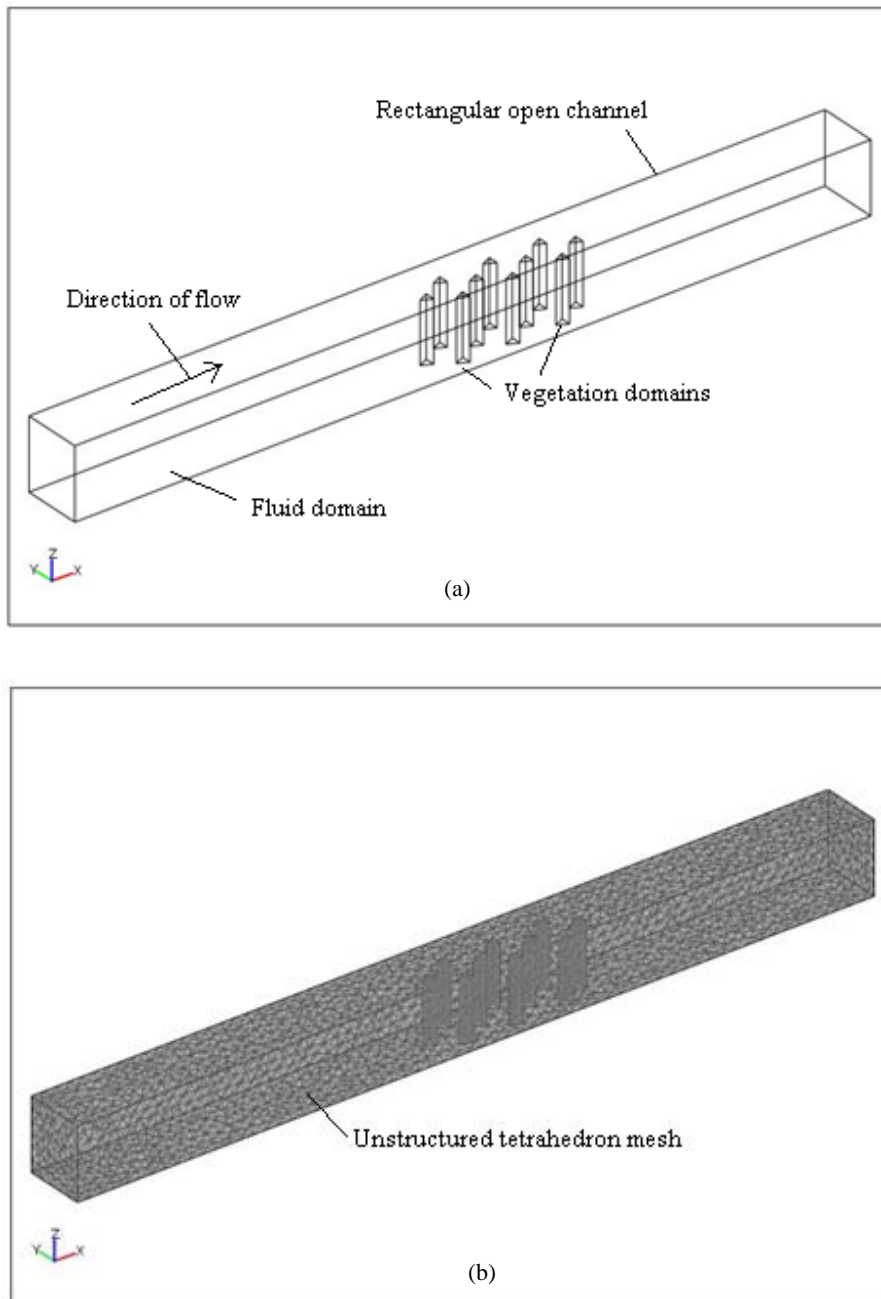


Figure 1: a) Model of channel with vegetation b) Domain discretization

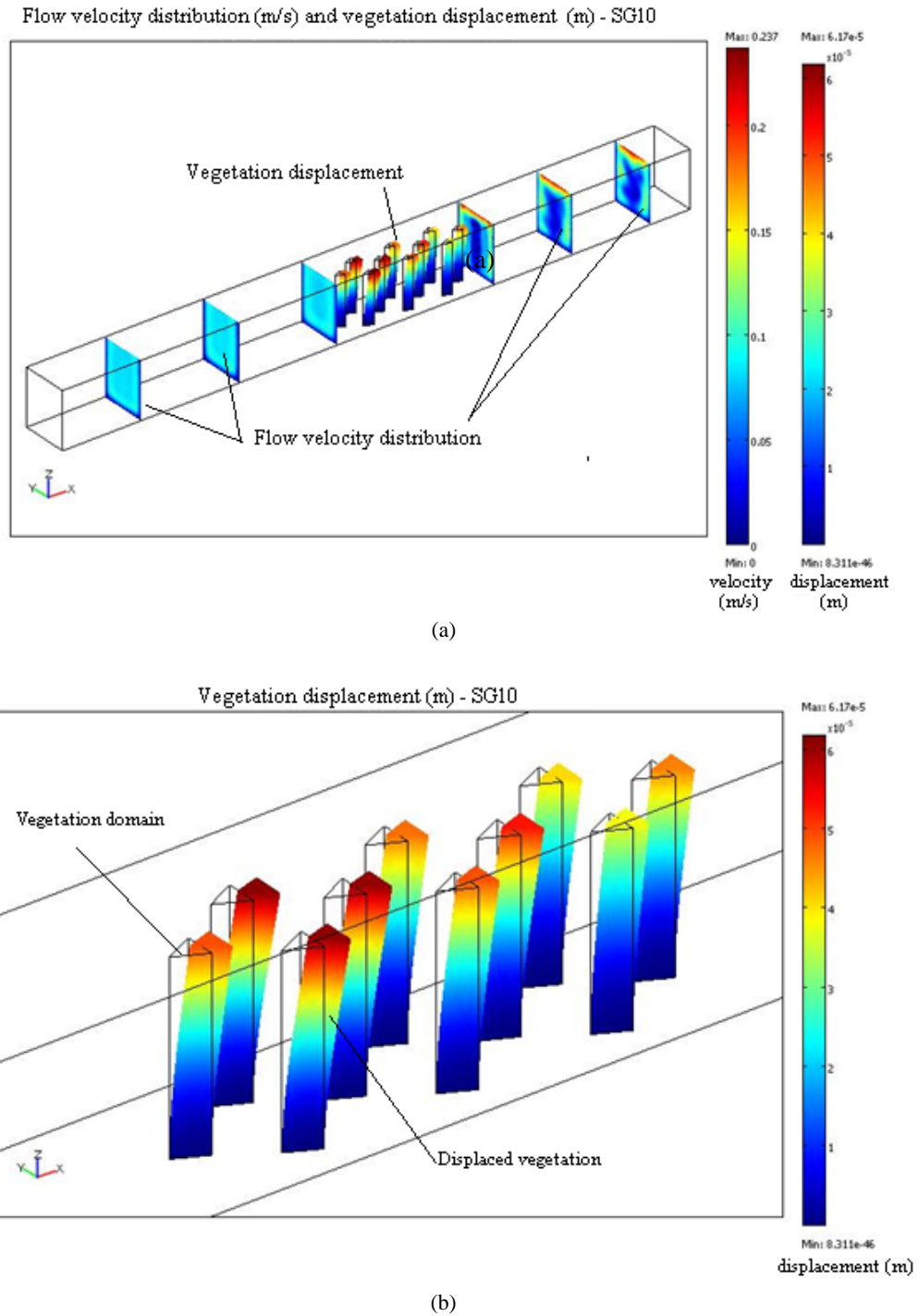
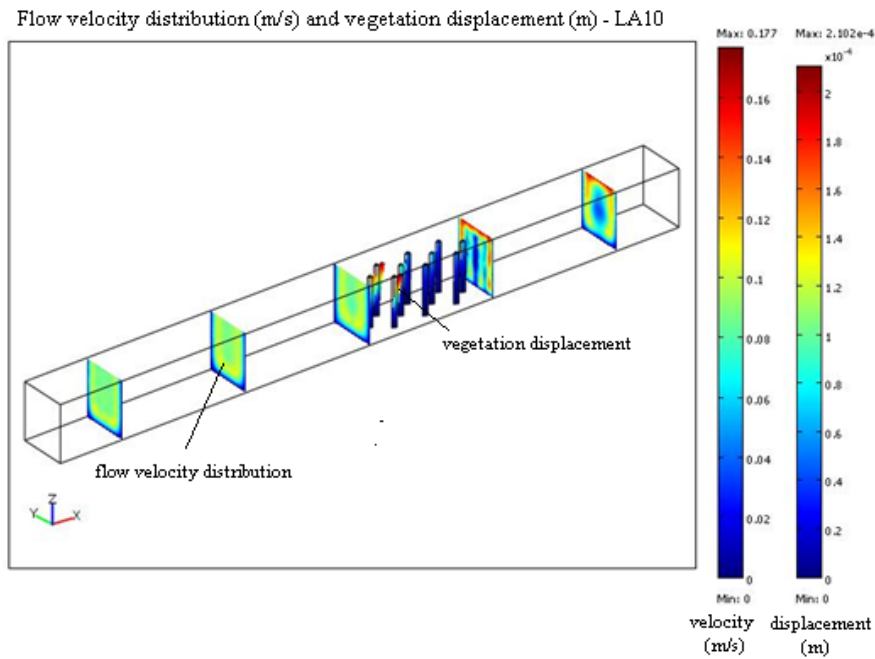
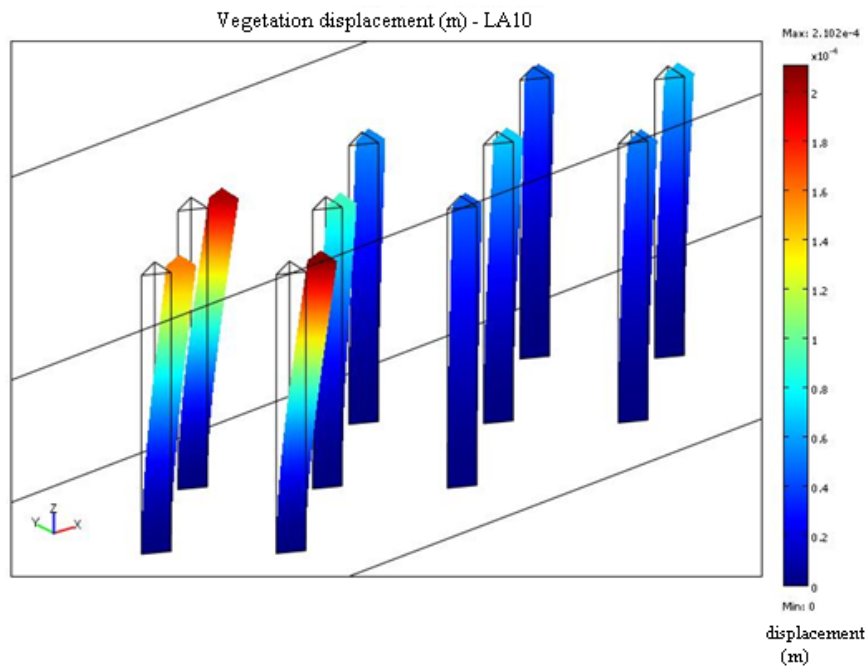


Figure 2: Output of numerical modeling for SG10-1 a) Cross sectional flow velocity distribution along the channel and vegetations displacement b) Vegetation displacement (Enlarged)



(a)



(b)

Figure 3: Output of numerical modeling for LA10 a) Cross sectional flow velocity distribution along the channel and vegetations displacement b) Vegetation displacement (Enlarged)

The profiles of the velocity in both simulations show good agreement with theoretical velocity profile, in which the velocity is zero at the channel boundary and maximum at the water surface. Flow velocity at the upstream of the vegetation show a core of high velocity at the centre of the channel. Inside the vegetation canopy and at the downstream of the vegetation, the regime of high velocity moved to the flow layer above the vegetation canopy. The pattern of vegetation deflection shows agreement with what would be expected from cantilever beam theory. The displacement is 0 at the fixed bottom of the vegetation and maximum at the free end. The deflection of *Scirpus Grossus* is found to be much smaller than that of *Lepironia Articulata* and the result is consistent with the modulus of elasticity of the vegetations. *Scirpus Grossus* has higher modulus of elasticity compared to *Lepironia Articulata*.

Figure 4 and figure 5 show the comparison between the velocity profiles calculated using the numerical models and the velocity profiles measured experimentally, for *Scirpus Grossus* and *Lepironia Articulata*, respectively. The graphs show the velocity variation with flow depth (z-direction) at the centre of channel cross section (y-direction) and at different locations along the channel (x-direction). Upstream of the vegetation ($0 < x < 3$ m), the pattern of velocity distributions is almost constant. The velocity profiles show logarithmic distribution with zero at the bottom of the channel and maximum at the water surface. In the vegetation region and at the downstream of the vegetation ($3 \text{ m} < x < 6$ m), the velocity profile shows three apparent layers with small velocity and large velocity gradients near the bed, almost constant velocity inside the vegetation canopy and high velocity above the vegetation canopy which becomes vertical towards the water surface. The results show similar pattern to those obtained by other researchers [1, 9-10, 19].

As mentioned earlier, the numerical models were solved using two different boundary conditions at the fluid-vegetation interface that are 'no slip' (SG10-1 and LA10-1) and 'velocity' (SG10-2 and LA10-2). Figure 4 and Figure 5 also show the comparison of the velocity profiles obtained using these two boundary conditions. At the upstream of the vegetation ($0 < x < 3$ m), the application of 'no slip' or 'velocity' boundary conditions gave no effect to the velocity profiles and the results of numerical models show good agreement with the results of experiment. However, at the vegetation region ($3 \text{ m} < x < 6$ m), the application of 'velocity' boundary condition at the fluid-vegetation interface gave velocity profiles that fit better to the experimental results compared to that of with the 'no slip' boundary condition. 'No slip' boundary condition implies no flow can pass through the vegetation domain but in actual conditions, flow can still pass through the vegetation stems, so the application of 'velocity' boundary conditions is more appropriate.

The variation of Manning roughness coefficient (Manning n) along the channel is shown in Figure 7 for SG10 and Figure 8 for LA10. The Manning n at a particular section is calculated using Manning formula and the mean velocity measured or computed at that particular section. From the figures, it can be seen that the Manning n calculated using the numerical models results agrees well with that of experiments. The Manning n at upstream and downstream of the vegetation is almost constant but at the vegetation region, the Manning n increased quite significantly due the resistance imposed by the vegetation.

Table 2 gives the difference in term of mean absolute error (MAE) between the computed results by means of numerical models with the measured results obtained experimentally. The differences are very small showing that the numerical model can be reliably used to estimate velocity and Manning n in a channel with flexible submerged vegetation.

CONCLUSIONS

The results show that the numerical models can be reliably used as an alternative to experimental approach in estimation of flow velocity and resistance in channels with flexible vegetation. All simulations results show good agreement not only with the experimental results but also with theory and others researches findings. This proved that the combination of Navier Stokes equation, stress-strain relationship and ALE algorithm is suitable for simulating flow and flexible vegetation interaction. Numerical models is just a tool to facilitate the analysis, the success of it depends on the correct input of boundary conditions and mesh specifications as well as the right choice of numerical solution technique.

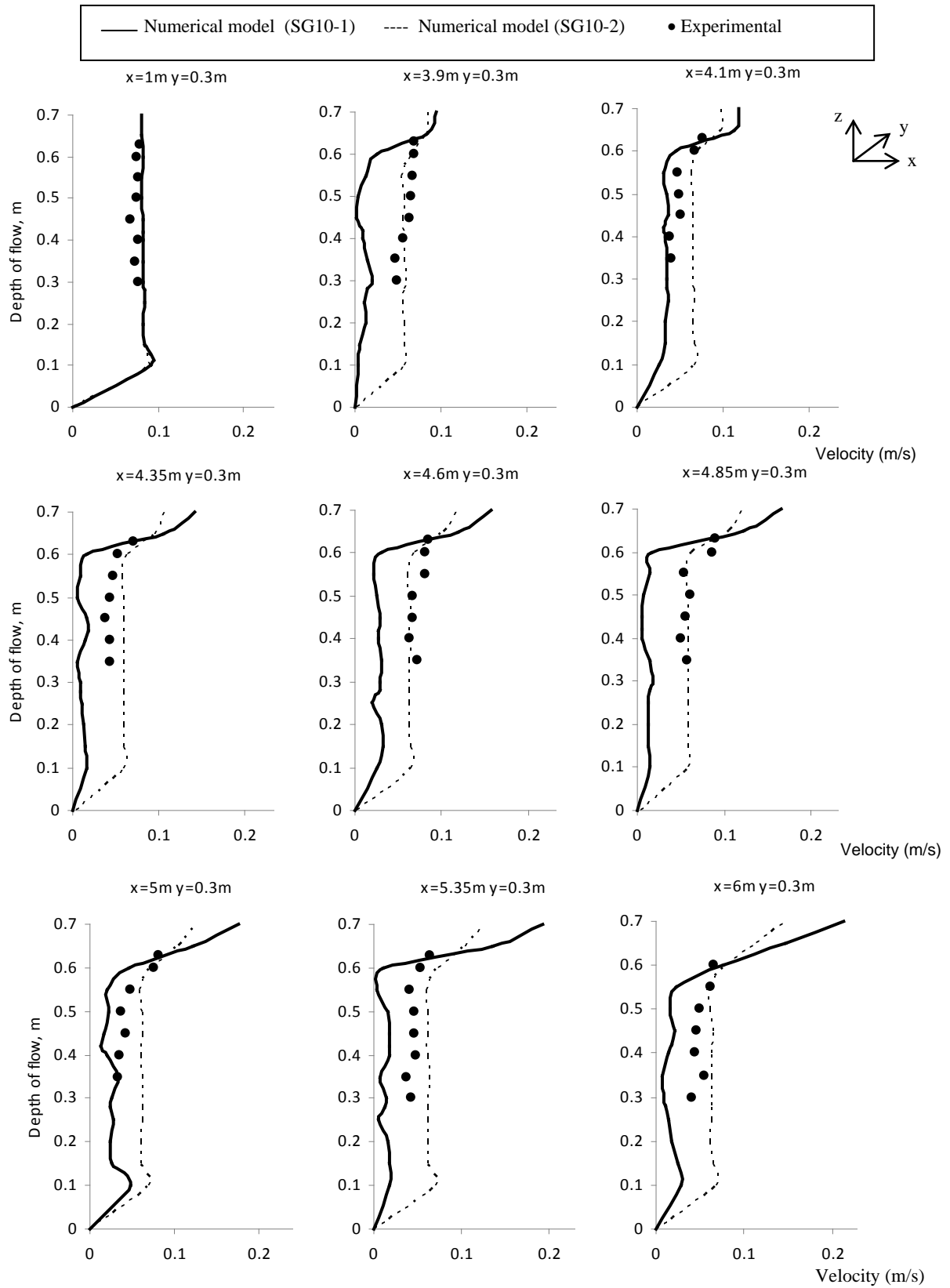


Figure 4: Vertical velocity profiles along the channel - Scirpus Grossus

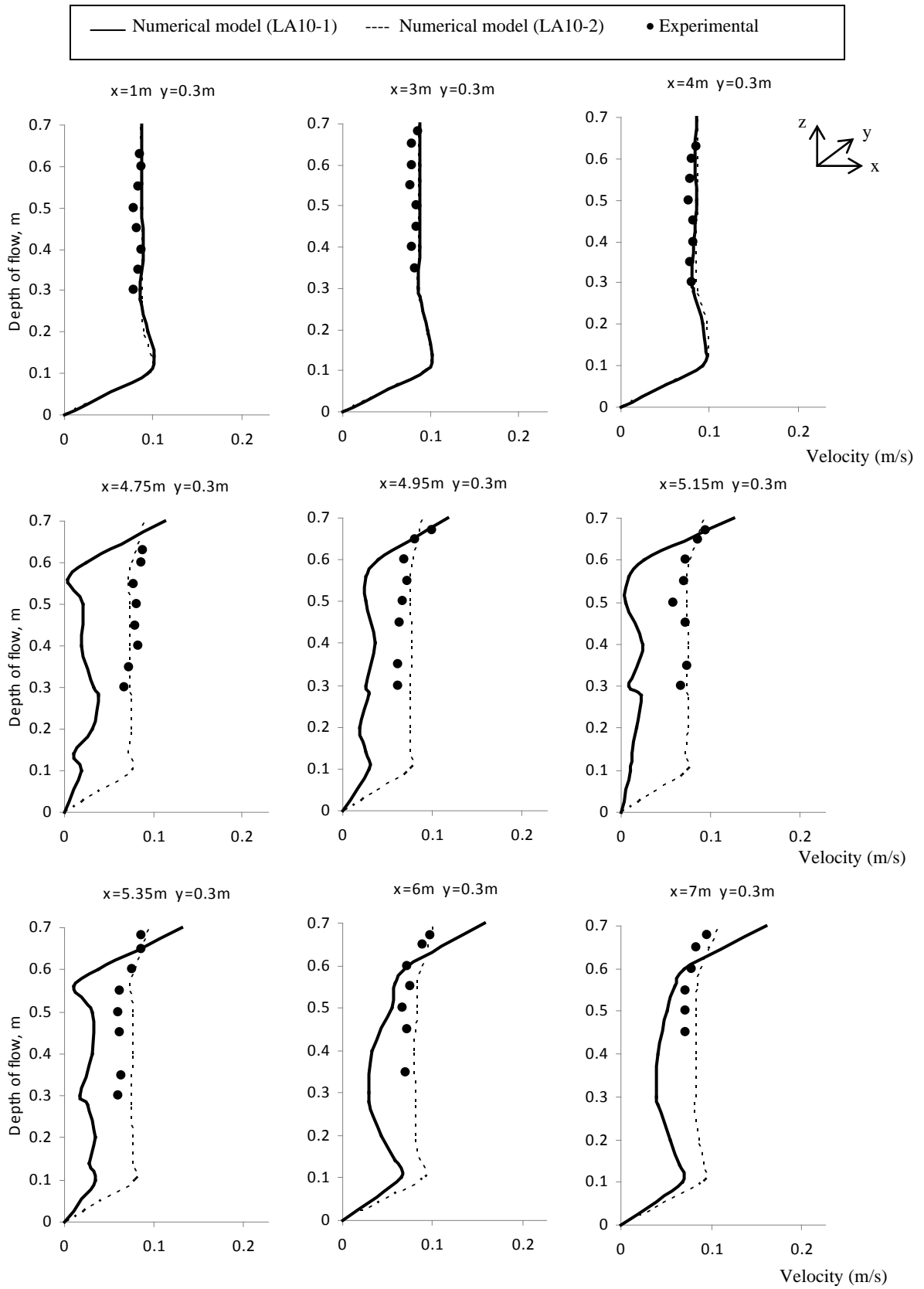


Figure 5: Velocity profiles along the channel - *Lepironia Articulata*

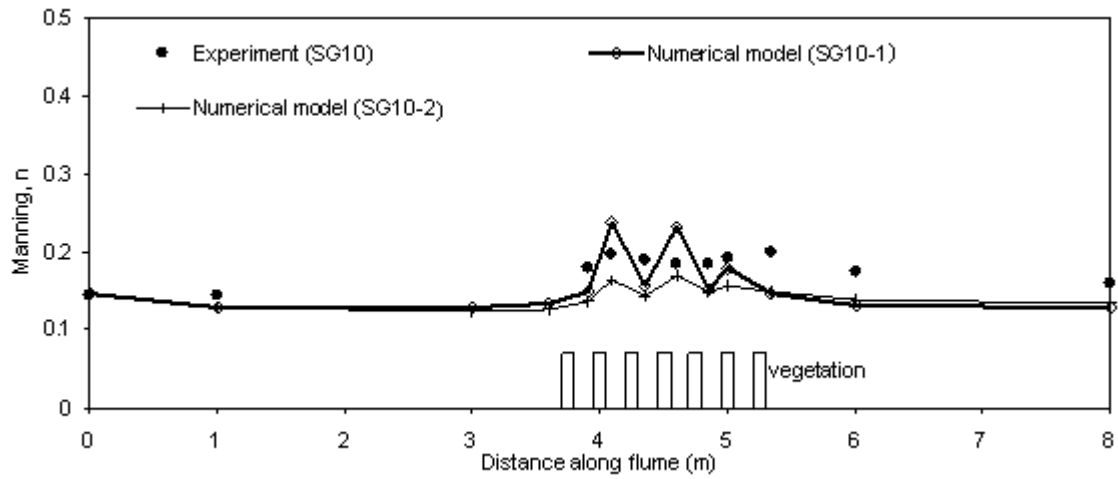


Figure 6: Manning n along the channel – Scirpus Grossus

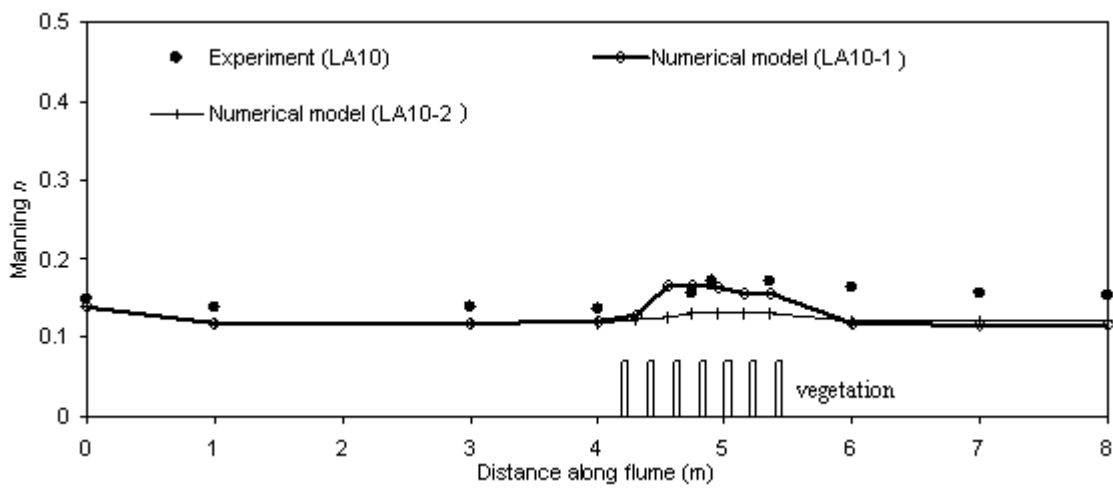


Figure 7: Manning n along the channel – Lepironia Articulata

Table 2: MAE between computed and measured mean velocity and Manning n

Vegetation	Velocity	Manning n
SG10	0.02	0.03
LA10	0.02	0.02

ACKNOWLEDGMENTS

The research was funded by Ministry of Science, Technology and Environment, Malaysia through Sciencefund Grant (Project No: 04-01-04-SF0575).

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