

## NUMERICAL AND FIELD STUDY ON TRIANGULAR SHELL FOOTING FOR LOW RISE BUILDING

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### ABSTRACT

*This paper describes a study on the performance of triangular shell footings using finite element and field model test. Two shapes of triangular shells were studied, namely the 'upright' triangular and inverted triangular shell. Both the finite element analyses and field tests showed that the inverted triangular shell had better load carrying capacity compared with the 'upright' triangular shell, due to its better contact area. The triangular shell in turn proved to be more efficient in carrying load compared with the conventional flat strip footing. The load carrying capacity of shell footing was also found to increase with the increase of shell angle and thickness. The load carrying capacity of shell footing was found to increase by around 15 % when the shell thickness was increased from 10 cm to 15 cm, and increase by 20% with the increase of the shell angle from 26 to 45 degree.*

**Keywords:** field test, finite element analysis, low rise building, shell footing

### INTRODUCTION

The concept of shell footing is not new in foundation design, considering past constructions using inverted brick arch foundation. The use of inverted brick arches as foundation or footing has been in practice in many parts of the world for a long time. Shells in modern foundation engineering are however relatively new. Shell footings have been found to be economical foundations in areas having high material to labor cost ratio (1, 2). Shell footing has greater load carrying capacity compared with flat shallow foundations. Moreover, shells are essentially thin structures, thus structurally more efficient than flat structures. This is an advantage in situations involving heavy super structural loads to be transmitted to weaker soils.

Shell footing in foundation engineering however is limited to a few geometries, such as conical, pyramidal, hyperbolic, spherical and triangular footings. These footings are shown in Figures 1 – 5 respectively. The conical shell (Figure 1) is the simplest form of shell, which can be employed in foundation engineering due to its singly curved surface. Due to its circular plan, the use of conical shell footing is restricted to an isolated footing only.

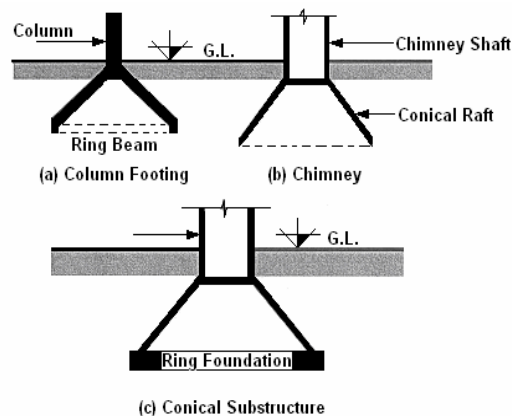


Figure 1: Typical detail of conical footing.

A pyramidal shell (Figure 2) is a combination of four inclined trapezoidal plate elements. Since the pyramid can be portrayed as square or rectangular in plan, multiple units of pyramidal shells can be jointly integrated to act as combined or raft foundation.

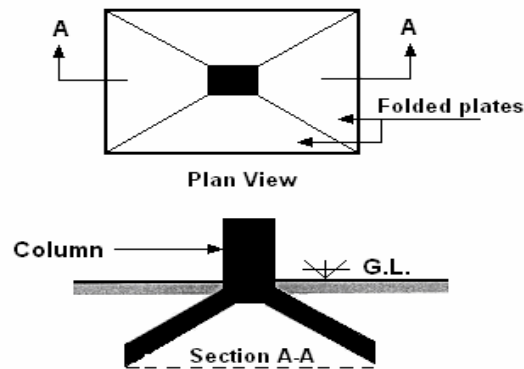


Figure 2: Typical detail of pyramidal footing.

The hyperbolic paraboloid (hypar) shell (Figure 3) is a doubly curved anticlastic shell, which has translation as well as ruled surfaces. This footing has a potential to be employed in a wide range of applications in foundation engineering.

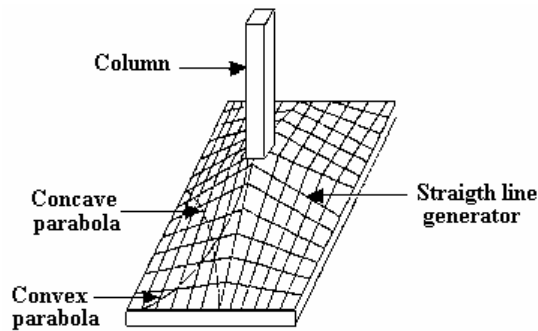


Figure 3: Typical detail of hypar footing.

Spherical shells (Figure 4) do not posses straight-line property, which makes its construction more complex. It can only be used as an isolated footing.

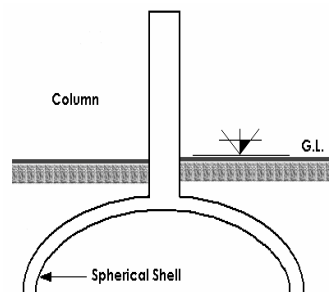


Figure 4: Typical detail of spherical footing.

For footings for mass housing using load-bearing walls constructed from such building system as the Putra block building system developed by the Housing Research Centre (HRC) of Universiti Putra Malaysia, triangular strip type shell footings might be the only possible solution (3, 4). This type of footing can be cast in small length panels and easily placed and connected at the location. A provision for pre-cast or cast in situ beam should be there to avoid minor differential settlement and to provide a levelled base for block masonry. The system requires a special footing at each corner of the wall. Sand can be poured and compacted after joining each panel and forming a wall footing, but before placement of beam. Another possible option might be to cut the soil in the same profile with smaller dimensions than the shell footing, place the pre-cast footing on it and fill up the gap with cement-sand slurry with controlled pressure to avoid uplifting of panels. The suggested system has potential to be installed quickly and safely on weak soil conditions.

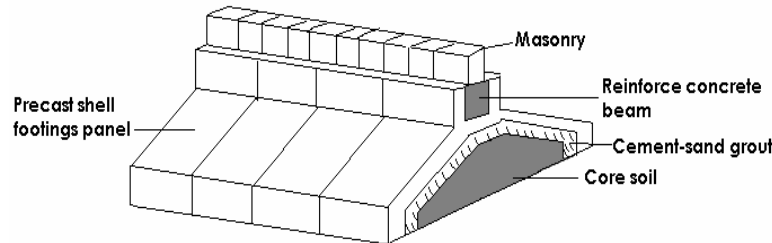


Figure 5: Pre-cast triangular shell footing for load bearing wall.

A number of experimental and theoretical investigations have reported the evaluation of the structural strength and behaviour of the shell structure, such as membrane stresses, bending moments, shear forces, and deflections. For theoretical analysis, mathematical formulations, namely finite difference technique and finite element analysis were utilized. In some studies, linear Winkler and Pasternak soil model was used to simulate the soil behaviour under different types of shell foundations. In the few studies, the distribution of the soil contact pressure on shell footing was examined. The results indicated a non-uniform contact pressure distribution along the soil-shell interface. However, the structural design of shell foundation is currently based on membrane theory, in which the soil contact pressure distribution is assumed to be uniform (5, 6, 7 and 8).

The ultimate strengths of the shell footings were also investigated both experimentally and theoretically; and comparisons were made with conventional flat footing. All studies reached the same conclusion concerning the saving achieved in the construction materials and the good structural performance of the shell footing. The findings of these investigations have a direct impact on the construction cost of shell footings as compared to the conventional flat counterparts (9). Abdel-Rahman (10), Hanna and Abdel-Rahman (11, 12) reported experimental results on conical shell footings on sand for plain strain condition. Maharaj (13) conducted a finite element analysis for conical shell footing to study the effects of increasing soil modulus.

Huat and Mohammed (14) described a study on the interaction between the shell footing and soil using a 2D non-linear finite element (FE) analysis program (PLAXIS). The effects of adding edge beams at the bottom of the footing, and depth of embedment of the footings, on the load carrying capacity of the footing are also investigated. They found that shell footing has better load carrying capacity compared with the conventional slab/flat footing of similar cross sectional area. The FE analysis also showed a reasonably good agreement with the laboratory experimental results. The effect of adding edge beams at the bottom of the shell footings has been found to be beneficial in increasing the load carrying capacity of the footing. The effect of increasing the embedment ratio is also found to increase the load carrying capacity of the shell footings.

This paper describes a study on the performance of triangular shell footings using a finite element program (LUSAS) in both 2 and 3 D. A field model test was also carried out. Two shapes of triangular shells were studied, namely the 'upright' triangular shell and inverted triangular shell. A parametric study was also carried out to examine the effect of shell thickness and shell angle on the load carrying capacity of the shell footing.

## VALIDATION OF THE FINITE ELEMENT MODEL

The triangular shell footings and soils were modelled and analyzed using a commercial finite element software, LUSAS. Non-linear Drucker Prager constitutive law was used to model the soil. The foundation was modelled with von Mises. Experimental results from earlier work of Abdel-Rahman (10) was used to validate the finite element modelling of the present study.

The geometry of the mesh for plain strain condition is symmetrical about the centreline, therefore only one half of the cross section passing through the axis of symmetry of the footing is considered. The nodes along the bottom and both sides of the section were considered as pinned supports, i.e., no movement was allowed in both vertical and horizontal directions, which called in the program as Standard Fixities. A smaller size element for the soil was selected in the vicinity of the footing where the variations of stresses and strains were expected to be more significant.

Figure 6 shows the typical generated mesh. Figure 7 shows the load – settlement curves of the FE analysis. Superimposed on Figure 7 is the load – settlement of the laboratory model test of Abdel-Rahman (10). In general there is a good agreement between the FE analysis and that of the laboratory model test. However, the results of the FE analyses are slightly higher than that of the laboratory experiments. This is inherent since the FE analysis was done in 2 dimensions while the experimental study was for a 3 D model.

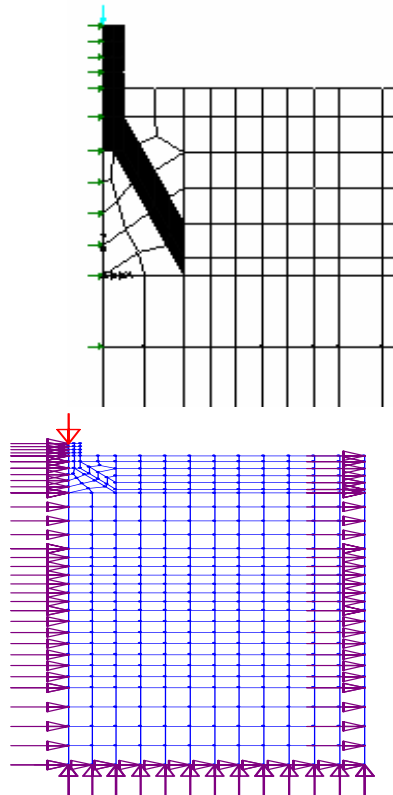


Figure 6: Typical generated mesh of the FE model.

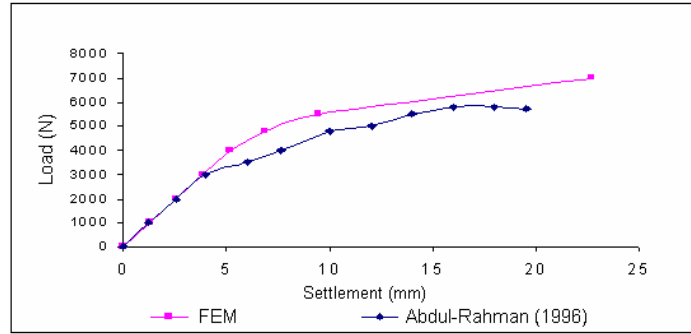
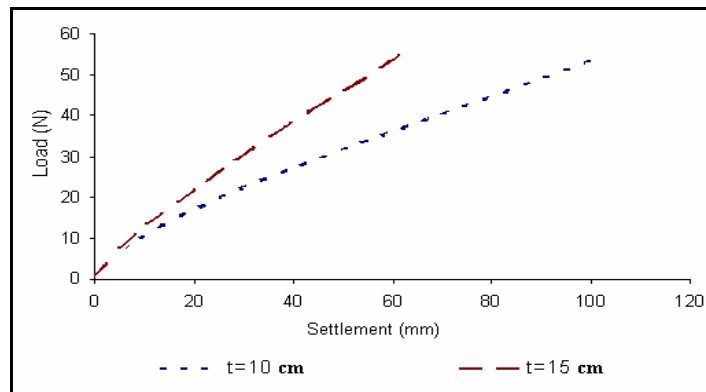


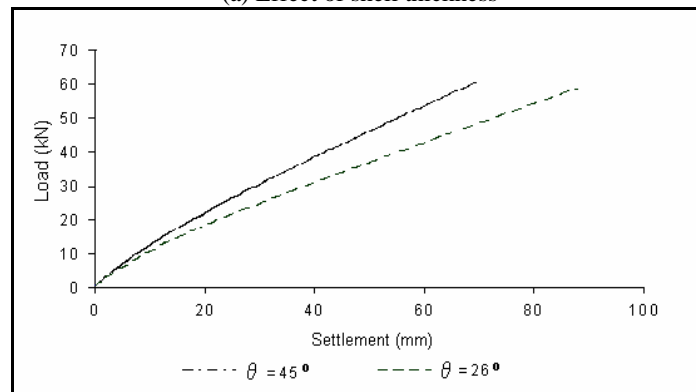
Figure 7: Load-settlement curves of the FE and laboratory model shell footing.

### PARAMETRIC STUDY ON THE EFFECT OF SHELL THICKNESS, SHELL ANGLE AND FOOTING SHAPE

Figure 8 shows the effect of shell thickness ( $t$ ) and shell angle ( $\theta$ ) on the load carrying capacity of the ‘upright’ triangular shell footing. The load carrying capacity of the triangular shell footings was found to increase with the increase of shell angle and thickness. The load carrying capacity of triangular shell footings was found to increase by around 15 % when the shell thickness increased from 10 cm to 15 cm, and increase by 20% with the increase of the shell angle from 26 to 45 degree.



(a) Effect of shell thickness



(b) Effect of shell angle

Figure 8: Effect of shell thickness ( $t$ ) and angle ( $\theta$ ) on load carrying capacity of triangular shell footing.

An attempt was then made to study the effect of inverting the triangular shell footing as shown in Figure 9. The hypothesis was that the load carrying capacity of the shell would be increased with an increase in footing contact area.

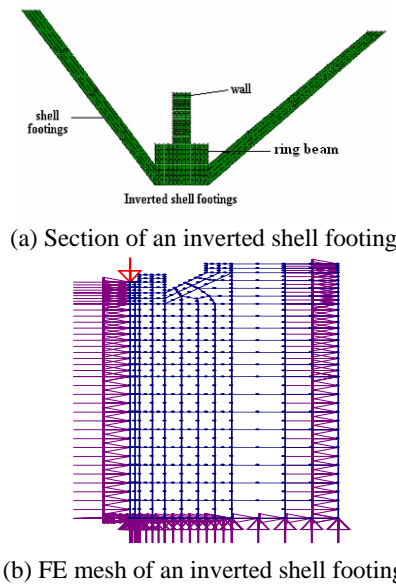


Figure 9: Inverted shell footing.

Based on the results obtained from the 2D FE analysis shown in Figure 10 below, the inverted triangular shell footings had higher load carrying capacity compared with the ‘upright’ triangular shell and conventional strip flat footing, by around 15% and 28% respectively.

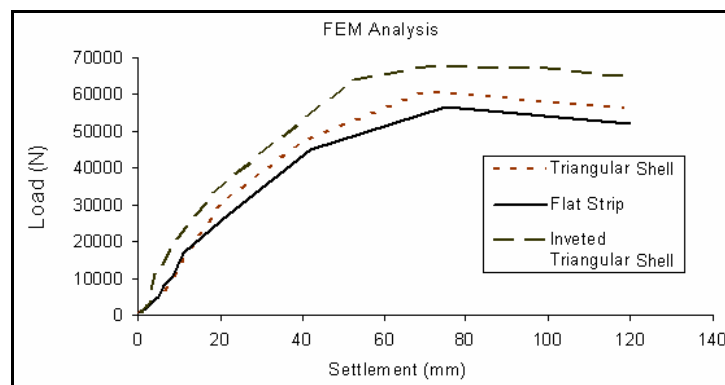


Figure 10: Load carrying capacity of inverted triangular shell, ‘upright’ triangular shell and flat strip footing.

A parametric study was carried out to study the effect of shell thickness ( $t$ ) and shell angle ( $\theta$ ) on the load carrying capacity of the inverted shell footing. As in the case of the ‘upright’ triangular shell footing, the load carrying capacity of the inverted triangular shell footings were also found to increase with the increase of shell angle and shell thickness, as shown in Figure 11.

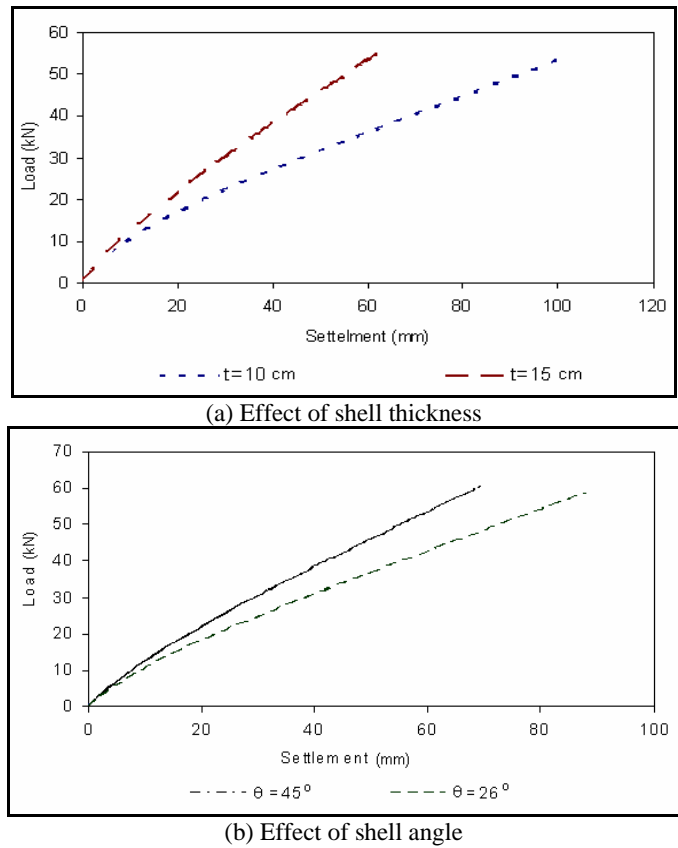


Figure 11: Effect of shell thickness (t) and angle ( $\theta$ ) on load carrying capacity of inverted triangular shell footing.

A 3 D analysis was also performed for the case of the inverted triangular shell footing. Consistent with the results of model study from the literature, the 3 D model (as shown in Figure 12) gave a slightly higher load carrying capacity for the shell footing.

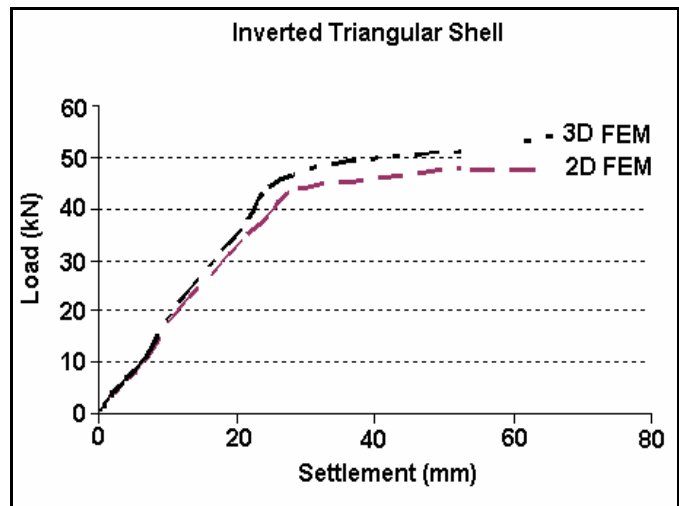


Figure 12: 3D analyses of inverted triangular shell footing.

## FIELD TEST

Field tests comprising three types of footings, namely a conventional flat footing, an 'upright' triangular shell and an inverted triangular shell footing were carried out. Figure 13 shows the experimental set up. Data obtained from the above mentioned parametric study was used to obtain an optimum cross section of inverted and 'upright' triangular shell footings. The model footings were monitored using load and pressure cells. Each footing was 1 m by 1 m in plan area.



(a) 'Upright' triangular shell footing.



(b) Inverted triangular shell footing.



(c) Instrumentation - Pressure cell



(d) Load cell



(e) Loading and testing.

Figure. 13: Field model test set-up and instrumentation.

Figure 14 shows the measured load settlement data. As expected, the inverted triangular shell footing showed higher load carrying capacity compared with the 'upright' triangular and conventional flat strip footing.



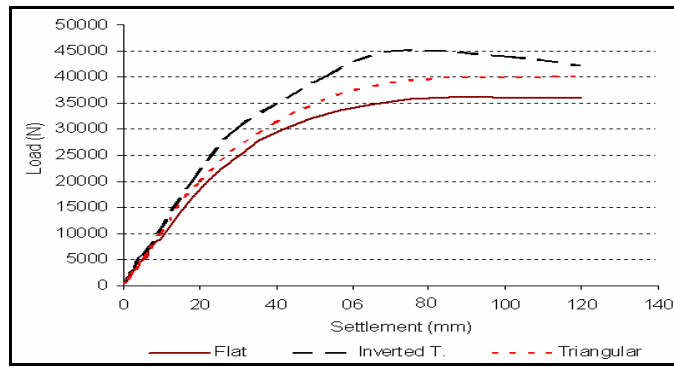
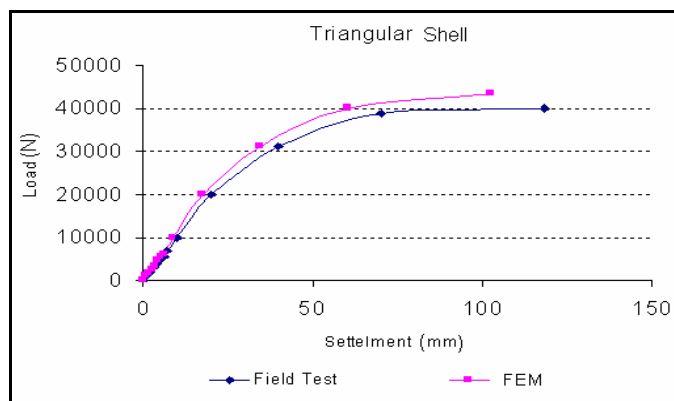
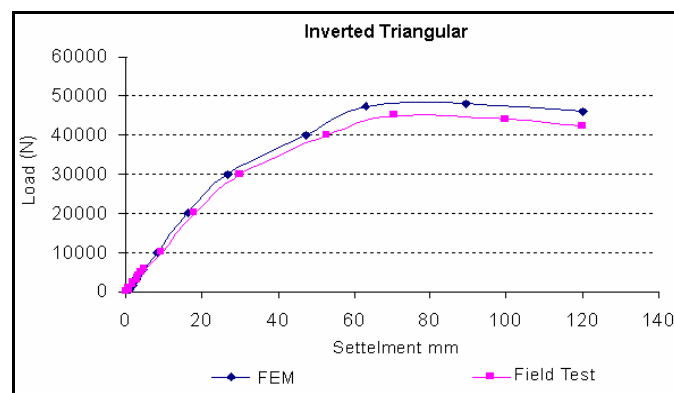


Figure 14: Measured load-settlement data of the field model footings.

Figure 15 shows the comparison of the field test with the 2D FE analysis. The comparison was apparently satisfactory for both cases of ‘upright’ triangular and inverted triangular strip shell footing. Figure 16 shows the contact pressures measured with pressure cells installed at various locations beneath the model footings. The triangular shell footing was found to exhibit higher stress concentration in the edge part of footing. However, for the inverted triangular shell, stress was better distributed over the contact area of the shell. The vertical stress of the inverted triangular shell footing was less than the ‘upright’ triangular shell footing by around 20 %.



(a) Triangular shell



(b) Inverted triangular shell

Figure 15: Field test results and 2D FE analysis.

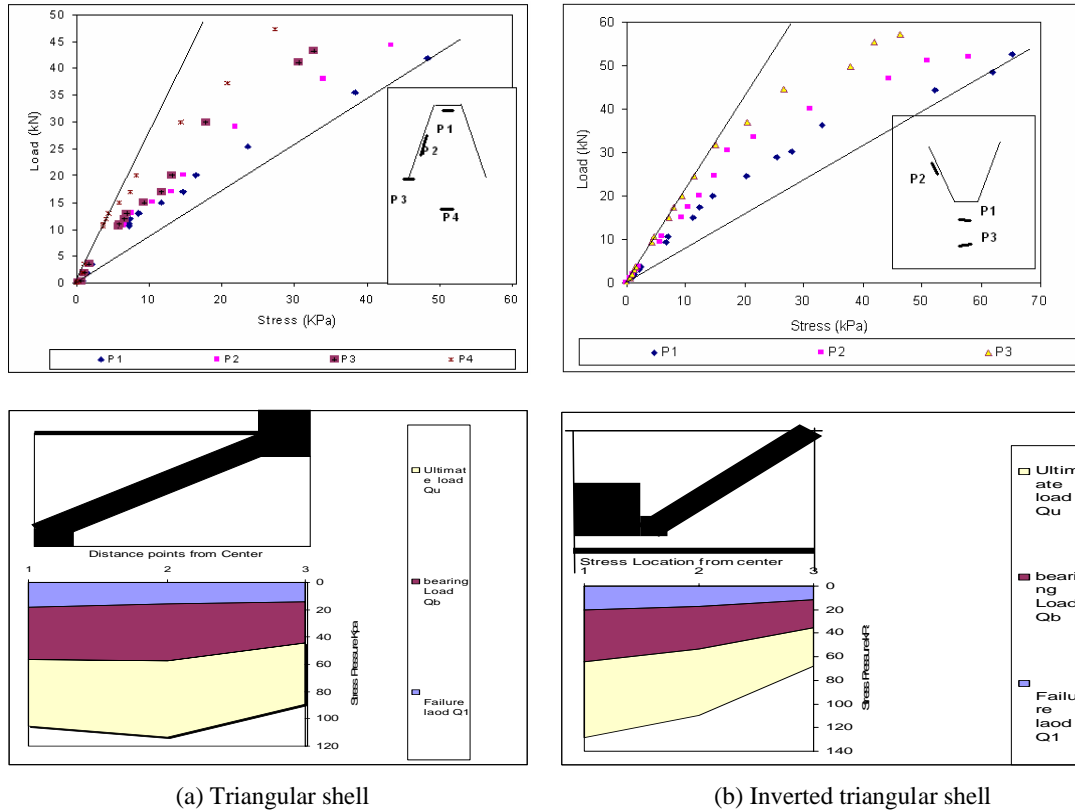


Figure 16: Stress distributions beneath the shell footings.

**CONCLUSIONS**

The 2 and 3 D finite element modelling using the computer software LUSAS, simulated the behaviour of the triangular shell footings with acceptable accuracy. Drucker Prager was used to model the soils, whilst von Mises was used to model the footing. The Finite Element analysis showed that the inverted triangular shell footings had higher load carrying capacity compared with other strip footings. The load carrying capacity of the inverted triangular shell footing was about 15% and 28% higher than the ‘upright’ triangular shell footing and conventional flat footing respectively. The load carrying capacity of shell footings was found to increase with the increase of shell angle and shell thickness. The load carrying capacity of shell footings was found to increase by around 15 % when the shell thickness increased from 10 cm to 15 cm, and increased by 20% with the increase of the shell angle from 26 to 45 degree. From the parametric study, an optimum cross section of inverted and triangular shell footings was identified and used for the field experiments. The results of the field model tests, as in the case of the FE analysis, showed that the inverted triangular shell footings had higher load carrying capacity compared with other strip footings (‘upright’ triangular and conventional flat strip). The measured contact pressure showed that triangular shell footing exhibited high stress concentration in the edge part of shell footing. However, for inverted triangular shell, stress was better distributed over the contact area of the shell. Vertical stresses of the inverted triangular shell footing was found to be lower than the stresses obtained for the triangular strip shell footings, by around 20 %.

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