NEW SATELLITE ATTITUDE CONTROL STRUCTURE USING THE GEOMAGNETIC FIELD

N. Mohd Suhadis and R. Varatharajoo
Department of Aerospace Engineering, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia
Email: gs16484@mutiara.upm.edu.my

ABSTRACT

The geomagnetic field is the main source of information considered when dealing with the magnetic attitude control system of a satellite. In this regards, the mathematical model of the geomagnetic field is described first in this paper. The simulation has been performed for the complex and simplified models, and the results show that the generated Earth's magnetic field values are almost identical for both models. For demonstration purposes, the simplified model is chosen to simulate the attitude control system of the small satellite. The performance of the system exhibits that the attitude steady state error is achieved with values of ±0.5 degrees. Thus, the magnetic attitude control system is a promising option for small satellites.

Keywords: *Geomagnetic field model, magnetic control system, momentum wheel unloading.*

INTRODUCTION

The existence of geomagnetic field is believed to have been generated within the earth. The theories about it can be reviewed in [**Error! Reference source not found.**] and [**Error! Reference source not found.**]. It can be concluded here that the three main sources that contribute to this field are the self excited dynamo effect in Earth's outer core, the electrical current flowing in the ionized upper atmosphere, and the induction by currents flowing within the Earth's crust. The former source that is also called the main field contributes over 95% of the total value. Therefore the derivation of the mathematical model is only from the main field and this model allows us to calculate the field value at any point near the Earth surface.

One of the important applications of the geomagnetic field is in the satellite attitude control system. The interaction between the geomagnetic field and the magnetic dipole moment generated within the satellite generates torque that can be used to control the satellite's attitude. This technique has been widely used because it is relatively lightweight, it presents a low power consumption and is extremely inexpensive compare to other methods of control [**Error! Reference source not found.-Error! Reference source not found.**]. In this paper, an appropriate geomagnetic field model will be selected (i.e., complex or simplified) in order to demonstrate an optional method of controlling a satellite.

GEOMAGNETIC FIELD MODEL

Complex Model

Since the existence of the geomagnetic field is practically arises from the motion of electrons, the mathematical model of the field derived is based on Maxwell's equations. However, the assumption has been made that the amount of current flowing across the boundary between the earth and its atmosphere is almost null and this implies that at the earth surface the curl of the vector **B** is zero, thus the vector field can be written as the negative gradient of a potential function as follows,

$$\nabla \times \mathbf{B} = 0 \tag{1}$$

$$\mathbf{B} = -\nabla V \tag{2}$$

In this case, the divergence of magnetic dipole vector field is considered zero because the magnetic flux directed inward the south-pole is equal to the flux outward the north-pole.

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

The combination of Eqs. (2) and (3) satisfies the Laplace equation that can be written as,

$$\nabla^2 V = 0 \tag{4}$$

The potential function V also can be expressed by a series of spherical harmonics [3],

$$V(r,\theta,\phi) = a\sum_{n=1}^{k}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left(g_n^m \cos m\phi + h_n^m \sin m\phi\right)P_n^m(\theta) \tag{5}$$

where a is the equatorial radius of the earth, (r,θ,ϕ) are the geocentric distance , co-latitude and east longitude from Greenwich (longitude is related to the right ascension α and can be expressed by $\phi = \alpha - \alpha_G$ where α_G is the right ascension of the Greenwich meridian or the sidereal time at Greenwich), (g_n^m, h_n^m) are Shcmidt normalized Gaussian coefficients and $P_n^m(\theta)$ is the Shcmidt normalized Legendre functions. The Shcmidt normalized Gaussian coefficients are determined from sets of measured data obtained from the satellites and ground observatories scattered around the world by a least-squares fit technique. These coefficients are revised in every 5 years because the geomagnetic field is constantly changing with time. Two main models of Gaussian coefficients that are available are the International Geomagnetic Reference Field (IGRF) and the World Magnetic Model (WMM) [**Error! Reference source not found.**]. The former is from the International Association for Geomagnetism and Aeronomy (IAGA) and the latter is a product of the U.S National Oceanic and Atmospheric Administration (NOAA) in collaboration with the British Geological Survey (BGS).

Thus, the general solution of the potential function of V expressed in spherical coordinate system can be written as follows,

$$\begin{aligned} B_r &= -\frac{\partial V}{\partial r} \\ B_\theta &= -\frac{1}{r}\frac{\partial V}{\partial \theta} \\ B_\phi &= -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} \end{aligned} \tag{6}$$

Simplified Model

The mathematical model of the geomagnetic field that has been described in the previous section can be written into a much simpler form with the assumption that the effects of Earth rotation and orbit precession are negligible for the duration of simulation. The geomagnetic field expressed in the local vertical local horizontal (LVLH) coordinate system is [13]

$$\begin{bmatrix} b_x(t) \\ b_y(t) \\ b_z(t) \end{bmatrix} = \begin{bmatrix} B_x \cos(\omega_0 t) \\ -B_y \\ B_z \sin(\omega_0 t) \end{bmatrix} \tag{7}$$

where the time t is measured from 0 at the ascending node of the orbit relative to the geomagnetic equator and B_x, B_y and B_z are constant values with the magnetic field's dipole strength μ_f equal to $7.9 \times 10^{15}\, Wb \cdot m^{-1}$.

$$\begin{aligned} B_x &= \left(\frac{\mu_f}{R_{Orbit}^3}\right)\sin(i) \\ B_y &= \left(\frac{\mu_f}{R_{Orbit}^3}\right)\cos(i) \end{aligned} \tag{8}$$

$$B_z = 2\left(\frac{\mu_f}{R_{Orbit}^3}\right)\sin(i)$$

Simulation

The Simulation has been carried out for both models with the orbital parameters as described in Table 1 using MATLAB 7.0. The results of the geomagnetic field values are expressed in the local vertical local horizontal coordinate system for both models and have been plotted together as in Figure 1. It can be seen that the both models generate similar results.

The $b_x(t)$ and $b_z(t)$ components of the field have a regular periodic behaviour while the $b_y(t)$ component has almost a constant value. This is because the x and z axis lies in orbital plane for the local vertical local horizontal coordinate system, and therefore, the components of the field along these axes vary with respect to the orbital motion. The $b_y(t)$ component is not influenced by the orbital motion because the y axis is normal to the orbital plane and it is only influenced by the rotation of the earth.

The values of geomagnetic field components are crucial for designing the attitude control gains. It has been shown that both the simulate models are similar. Therefore, the tuning of the attitude control gains can be based on the simplified model for the validation of the attitude control structure.

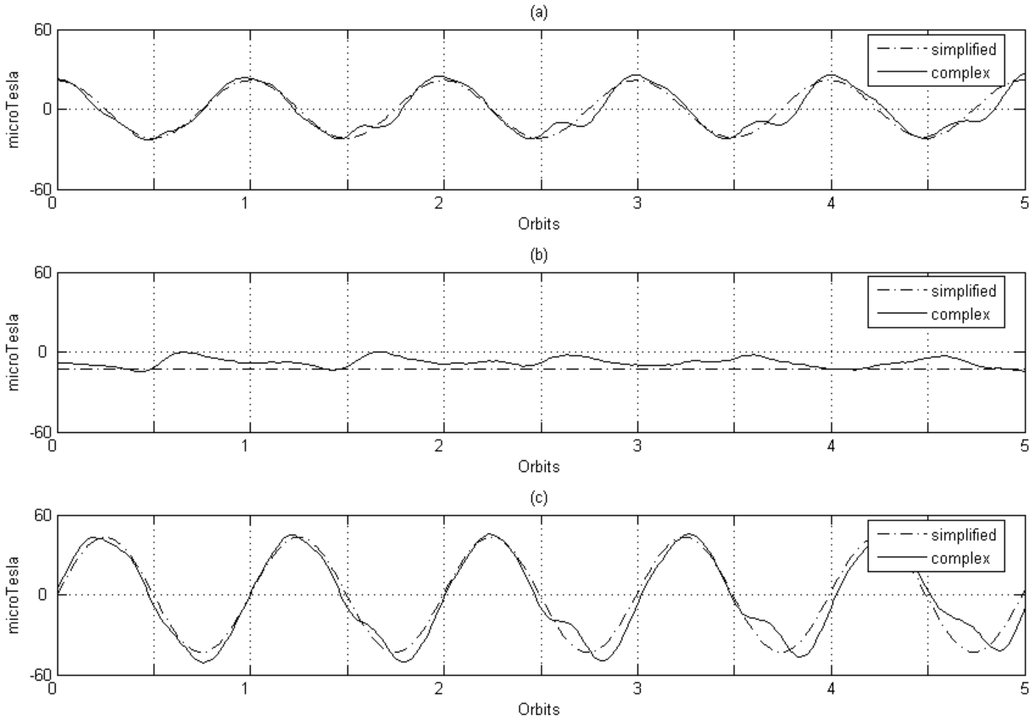

Figure 1: Comparison of (a) b_x , (b) b_y and (c) b_z components of Complex and Simplified Model of Geomagnetic Field

Table 1: Orbital Parameters

Parameter	Value
Altitude, h	$540\,km$
Inclination, i	$60°$
Eccentricity, e	1.68×10^{-4}
Right ascension of ascending node, Ω	$0°$
Argument of perigee, ω	$0°$
True anomaly, ν	$0°$
Simulation time, t	$28630\,s$ (5 orbits)
Epoch	1 April 2006

Orbital frequency, ω_0	$0.0011\, rads^{-1}$

SPACE MISSION

Satellite Configuration

To use the geomagnetic model for the satellite magnetic control system, the small satellite design is selected as depicted in Figure 2. The momentum wheel along the *-y* axis is used to provide gyroscopic stiffness along this axis. The two magnetic torquers are assigned for different tasks, where along the *+x* axis, it manages the bias momentum provided by the momentum wheel within allowable value. Whereas along the *+y* axis, it is dedicated to control the attitude rate of the *x* and *z* axes. Lastly, the gravity gradient boom is used to increase the moment of inertia along the x and *y* axes axis, thus, giving the gravity gradient stabilization along the *z* axis [**Error! Reference source not found.**]. The linearized dynamic equation of motion of the satellite can be described as follows

$$\begin{aligned} I_x\ddot{\phi} + \left[h_{wy} - \left(I_x - I_y + I_z\right)\omega_0\right]\dot{\psi} + \left[4\left(I_y - I_z\right)\omega_0^2 + h_{wy}\omega_0\right]\phi &= T_{cx} + T_{dx} \\ I_y\ddot{\theta} + 3\left(I_x - I_z\right)\theta\omega_0^2 &= T_{cy} + T_{dy} + \dot{h}_{wy} \\ I_z\ddot{\psi} - \left[h_{wy} - \left(I_x - I_y + I_z\right)\omega_0\right]\dot{\phi} + \left[\left(I_y - I_x\right)\omega_0^2 + h_{wy}\omega_0\right]\psi &= T_{cz} + T_{dz} \end{aligned} \tag{9}$$

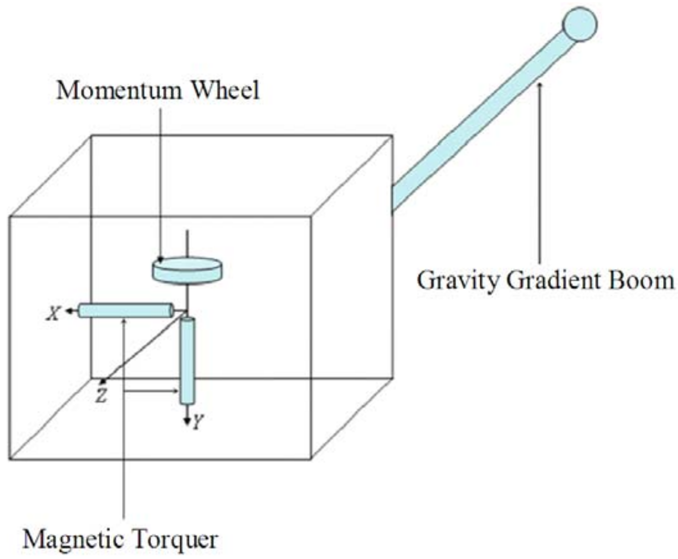

Figure 2: Satellite Configuration

Attitude Control Law

The control torque direction is normal to both the geomagnetic field vector and the satellite's magnetic dipole

$$T = M \times B \tag{10}$$

The formulation for the attitude control structure is based on Ref. [11]. However, the final control structure is modified in this work as follows

$$\begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} = \begin{bmatrix} 0 & b_z \\ -b_z & \dfrac{-b_x b_z}{b_y} \\ b_y & 0 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & 0 & A_{23} \end{bmatrix} \begin{bmatrix} h_{ox} \\ \Delta h_w \\ h_{oz} \end{bmatrix} + \begin{bmatrix} b_z \\ 0 \\ -b_x \end{bmatrix} \begin{bmatrix} B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_z \end{bmatrix} \tag{11}$$

The first term on the right part of Eq. (11) is for controlling the attitude and momentum while the second part of the equation is for controlling the nutation. For the simulation, the geomagnetic field values $b_x(t), b_y(t)$ and $b_z(t)$ are generated using the simplified geomagnetic field model.

In-orbit External Disturbances

The orbit perturbation of small satellite in Figure 2 is caused by the external disturbance torques. In low earth orbit, the major sources of the disturbance torques are gravity gradients T_{gg}, magnetic fields $T_{Magnetic}$, aerodynamics drag T_{Aero} and solar radiation pressure T_{Solar}.

$$T_d = T_{gg} + T_{Magnetic} + T_{Aero} + T_{Solar} \tag{12}$$

It can be briefly described here that the gravity gradient disturbance torque exists from the variation of the earth's gravitational force over the asymmetric body orbiting. The magnetic disturbance torque is caused by the interaction between the satellite's residual magnetic field and the geomagnetic field while the aerodynamic disturbance torque is caused by the interaction between the upper atmosphere with the satellite surface. Finally, the solar radiation disturbance torque exists from the solar radiation particle that hit the satellite's surface. The strength of these torques vary with the altitude and are modelled as follows

$$T_{gg} = \frac{3\mu}{R_{Orbit}^3}\begin{bmatrix}(I_3 - I_2)\phi \\ (I_3 - I_1)\theta \\ 0\end{bmatrix} \tag{13}$$

$$T_{Magnetic} = DB_{LVLH} \tag{14}$$

$$T_{Aero} = F\left(c_{pa} - c_g\right) \tag{15}$$

$$T_{Solar} = F\left(c_{ps} - c_g\right) \tag{16}$$

These external disturbances can be modelled as the sum of a constant and harmonic quantity as follows

$$T_d = T_{Constant} + T_{Harmonic}\sin(\omega_0 t) \tag{17}$$

Satellite Attitude Control Performance

The attitude control simulation parameters and the orbital parameters are as in Table 2 and Table 1, respectively. This simulation has been carried out using MATLAB 7.0. The results of generated attitude pointing errors for roll and yaw axis are depicted in Figure 3. As it can be seen, the attitude pointing errors is ±0.5 degrees. The attitude performances comply with most small satellite missions.

Table 2: Satellite Configuration

Parameter	Value
I_x	$230\,kgm^2$
I_y	$240\,kgm^2$
I_z	$25\,kgm^2$
h_{wr}	$14\,Nms$
$M_{magnetorquer}$	$15\,Am^2$
T_{dx}	$12.8\times10^{-6} + 8.6\times10^{-6}\sin(\omega_0 t)\,Nm$
T_{dy}	$55\times10^{-6} + 55\times10^{-6}\sin(\omega_0 t)\,Nm$
T_{dz}	$12.8\times10^{-6} + 4.3\times10^{-6}\sin(\omega_0 t)\,Nm$

ω_n	$4\times10^{-4}\, rad\cdot s^{-1}$
Attitude Gains	$A_{11}=0$, $A_{12}=\omega_n^2$, $A_{13}=-0.002/B_y$, $A_{21}=-0.002$, $A_{23}=0$
Nutation Gains	$B_{21}=-1.2$, $B_{22}=1.2$

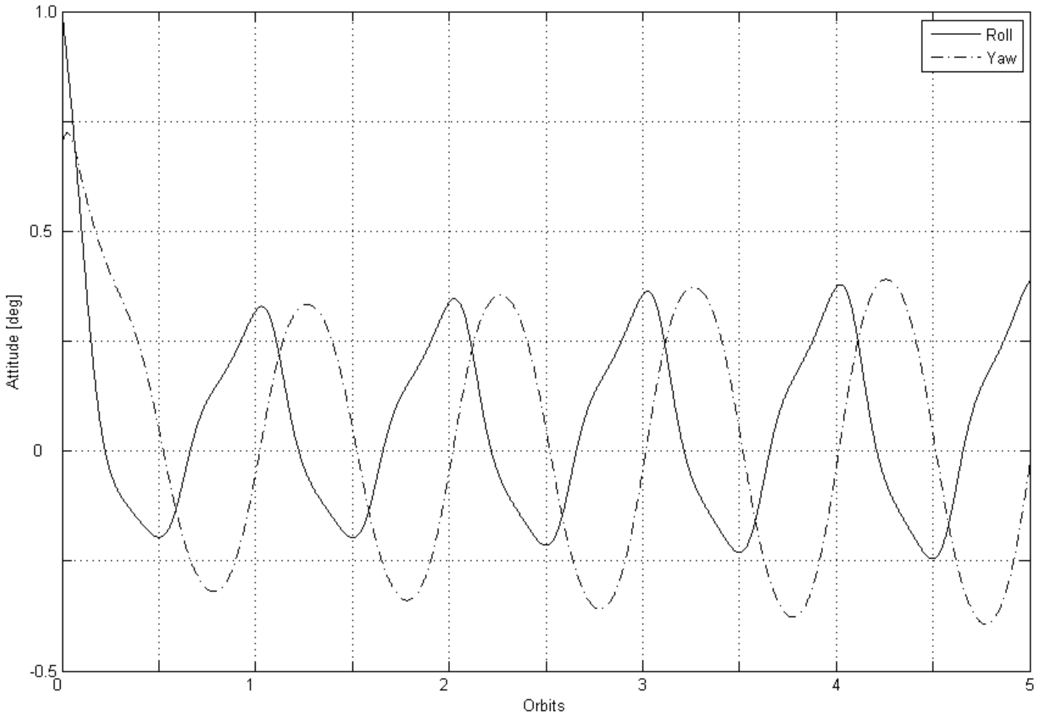

Figure 3: Attitude Pointing Error

CONCLUSION

In this paper the complex and simplified mathematical models of the geomagnetic field have been successfully simulated and compared. The cost in terms of the computational time can be saved by using the simplified model. For demonstration purposes, the simplified model has been chosen for the validation of the attitude control structure. The attitude control structure which is newly designed herein shows a promising in-orbit performance. The result shows that the satellite is stabilized within one orbit with an attitude accuracy of ±0.5 degrees. Finally, it can be concluded that the new attitude control structure is a potential control option in small satellites.

REFERENCES

[1] Campbell, W.H (2003) Introduction to geomagnetic fields. 2nd edition Cambridge University Press.

[2] Jacobs, J. A. (1994) Reversals of the earth's magnetic field. 2nd edition Cambridge University Press.

[3] Wertz, J. R. (1978) Spacecraft attitude determination and control. Kluwer Academic Publisher.

[4] Sidi, J. S. (1997) Spacecraft dynamic and control. Cambridge University Press.

[5] McLean, S., S. Macmillan, S. Maus, V. Lesur, A. Thompson and D. Dater (2004) The US/UK World Magnetic Model for 2005-2010, NOAA Technical Report NESDIS/NGDC-1.

[6] Silani, E. and Lovera, M. (2003) Magnetic spacecraft attitude control: a survey and some new results. Control Engineering Practice.

[7] Musser, K. L. and Ebert, W. L. (1989) Autonomous spacecraft attitude control using magnetic torquing only. Flight Mechanics Estimation Theory Symposium NASA.

[8] Leonard, B. S. (2002) NPSAT1 magnetic attitude control system. 16th Annual AIAA/USU Conference on Small Satellites

[9] Lovera, M. and Astolfi, A. (2004) Spacecraft attitude control using magnetic actuators. Automatica **40**: 1405 – 1414
[10] Lovera, M. and Astolfi, A. (2002) Global spacecraft attitude control using magnetic actuators. Proceedings of the American Control Conference, 1331 – 1335
[11] Steyn, W.H. and Hashida, Y. (2001) In-orbit Attitude Performance of the 3-Axis Stabilized SNAP-1 Nanosatellite, 15th AIAA/USU Conference on Small Satellites.
[12] Santoni, F. and Tortora, P. (2001) Magnetic Attitude Determination and Control of Inertial Pointing Small Momentum Bias Spacecraft, Advances in the Astronautical Sciences **108**(1): 607-625.
[13] Psiaki, M. L. (2001) Magnetic Torquer Attitude Control via Asymptotic Periodic Linear Quadratic Regulation, Journal of Guidance Control and Dynamics **24**(2):386-394.

NOMENCLATURE

B	= Geomagnetic field vector
b_x, b_y, b_z	= Geomagnetic field vector component described in LVLH coordinate system
B_r, B_θ, B_ϕ	= Geomagnetic field vector component described in spherical coordinate system
R_{Orbit}	= Satellite orbit
ϕ, θ, ψ	= roll, pitch and yaw
μ	= Earth gravitational parameter
I_x, I_y, I_z	= Moment of inertia
h_{0x}, h_{0z}	= Angular momentum
h_{wy}	= Wheel bias momentum
Δh_w	= Wheel momentum unloading
$\dot{h}_{wy}$	= Torque
T_{dx}, T_{dy}, T_{dz}	= Disturbance torques
T_{cx}, T_{cy}, T_{cz}	= Control torques
ω_n	= Nutation frequency
ω_x, ω_z	= Axis frequncies
D	= Residual dipole
F	= Force
c_{pa}, c_{ps}, c_g	= centre of aerodynamic pressure, the location of solar force, satellite's centre of gravity