COMPARISON OF DIFFERENT DWELL POINT POLICIES FOR SPLIT-PLATFORM AUTOMATED STORAGE AND RETRIEVAL SYSTEM

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ABSTRACT

In designing container terminals one have to consider the choice for a certain type of storage and retrieval equipment by performing a feasibility and economic analysis. The requirements for a more efficient container storage structure can be met by automated storage/retrieval system (AS/RS) because of its abilities of fully automating material handling and the potentials for high performance delivery. Stacker cranes used by the conventional AS/RS are not adequate for handling heavy loads such as sea container cargo. However, for such applications, to handle extra heavy loads (such as loads above 20 tons) at high speed, the split-platform AS/RS or SP-AS/RS for short is proposed by researchers. High lifting capacity enables the SP-AS/RS to deal with all the different types and sizes of containers which pass through the container terminal at the interchange point. In the case of travel time of split-platform AS/RS, only one perfect travel time model can be found in the previous researches, which is under stay dwell point policy. But it is not logical to select this travel time model as the base, without any analysis of other travel time models (under different dwell point policies). In this paper the authors develop two perfect travel time models under return to middle point and return to start point, dwell point policies. Models are validated by computer simulations and the results of different models are compared. The results show that stay dwell point policy outperforms other dwell point policies.

Keywords: *automated storage/retrieval system, split-platform AS/RS, travel time model, dwell point policy*

INTRODUCTION

What is SP-AS/RS?

Since its introduction in the mid-fifties containerized transportation has experienced a significant growth. Worldwide container trade is growing at a 9.5% per annum rate, and the U.S. rate is around 6%. It is anticipated that the growth in containerized trade continues as more and more cargo are transferred from break-bulk to containers [1]. By 2010, it is expected that 90 percent of all liner freight will be shipped in containers [2]. Every major port is expected to double and possibly triple its cargo by 2020. To deal with this steadily increasing proportion of cargo handled with containers, the capacity of ships has been extended up to 8000 TEU (twenty feet equivalent unit container). At the same time, there has been an increase in the importance of marine transport systems, including networks of ports and terminals. The docking time at these ports must be as small as possible to efficiently handle these big ships and as a result to satisfy customer's requirements and reduce costs [3]. A container terminal (or terminal in short) in a port is the place where container vessels dock on berths and unload inbound (import) containers (empty or filled with cargo) and load outbound (export) containers. The terminals have storage yards for the temporary storage of these containers [4]. In a conventional storage yard for containerized cargos, containers are stacked side by side, one on top of another (Figure 1). The major disadvantage of this stacking scheme is that the reshuffling procedure, which incurs additional unproductive moves, has to be performed in order to retrieve a container that is not at the top of the stack. Moreover, due to the strength constraint of containers, the stacking height is limited, which implies low floor-space utilization. An automated storage/retrieval system (AS/RS) is capable of providing random access to individual storage locations; it is also structurally possible to construct a high AS/RS structure [5, 6].

Structure and operation of the SP-AS/RS

Unlike stacker cranes, SP-AS/RS has one vertical platform (VP) for each rack and N horizontal platforms (HP) to serve N tiers of an AS/RS rack, (see Figure 2). The vertical platform provides the vertical link among different tiers of the AS/RS rack, whereas the horizontal platforms access the storage cells on a given tier. The vertical platform and the horizontal platforms may move independently and concurrently; and the separation of the mechanisms for vertical/horizontal movements also makes the platforms lighter and hence they can operate at a higher speed than the conventional design [5].

This separation also brings the potential benefits of higher handling rate, easier maintenance, and reduced downtime. The input/output (I/O) station is located at the ground level on one end of the rack. The I/O stations are the interface with the external system that carries loads to/from the AS/RS, and the hand-over stations are the locations where a loaded VP delivers the container to an empty HP or vice versa. The VPs transfer loads in between the I/O stations and the handover stations at any tier of the storage racks, whereas the HPs provide the horizontal connection from the hand-over stations to the individual storage cells. Such a system is capable of concurrent operations, that is, the VPs and the HPs can move independently and in parallel.

Figure 1: Conventional storage yard for storing container cargos at Imam Khomeini Port (western south), Islamic Republic Iran.

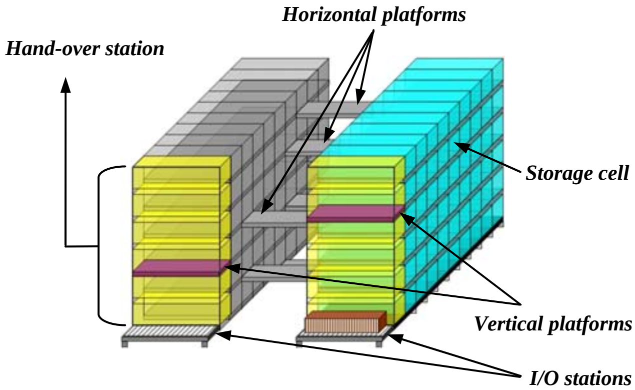

Figure 2: Structure of SP-AS/RS

Dwell point positioning

The dwell point in an AS/RS is the position where the S/R machine resides when the system is idle. The dwell point is selected such that the expected travel time to the position of the first transaction after the idle period is minimized. An effective dwell point strategy may reduce the response times of the AS/RS, since the S/R machine typically performs a sequence of operations following an idle period. Hence, if the first operation is advanced, then all operations within the sequence are completed earlier [7]. In this paper three prevalent dwell point policies for SP-AS/RS are used (Appendix A). These policies have been suggested in previous literature:

Return to middle (dwell point) policy: under this policy**,** the HP returns to middle of tier upon finishing a job and the VP returns to middle of handover station upon finishing a job**.**

Return to start (dwell point) policy: the VP returns to the I/O station and the HP returns to the handover station upon finishing a job [6].

Stay dwell point policy: the platforms remain where they are after completing a storage and retrieval operation [5], (The platforms will stay where they are at the end of each operation until they are required for another operation later [6]).

LITERATURE REVIEW

Literature in the area of dwell points for S/R machines

There is extensive research in the area of dwell points for S/R machines. Graves et al. [8] selected the dwell point of S/R machine at the input/output (I/O) station. They introduced the design, planning and control of warehousing systems as new research topics. For the dwell point specification problem, the following static dwell point rules in use were outlined by Bozer and White [9], although they provide no quantitative comparison of their performance,

1. Always position the S/R machine at the input station whenever idle.

2. Always position the S/R machine at a midpoint location in the rack whenever idle.
3. Position the S/R machine at the last storage location it visited after the completion of a single command storage task. Otherwise, position at the input station after the completion of either a single command retrieval or dual command cycle.
4. Position the S/R machine at the input station after the completion of single command storage. Otherwise, position at the output station.

Han et al. [10] have improved the performance of an AS/RS by sequencing the order of servicing the retrieval requests. In this case, the AS/RS is assumed to be throughput-bound and therefore is never idle. Many other papers have looked at various aspects of AS/RS design and operation [8, 11, 12]. Egbelu [13] proposed a framework for selecting the dwell-point location of the S/R machine. The author developed two formulations based on the relative likelihood that the next request was storage or retrieval. The first formulation uses an objective of minimizing the expected response time and the second one uses an objective of minimizing the maximum response time for an AS/RS. He then transformed these nonlinear programming formulations into linear programming problems that can be solved optimally. No discussion was included on the effectiveness of his framework. Park [14] shows the optimality of the strategy of Graves et al. [4], if the probability of the first operation after an idle period being a storage is at least 0.5.

Egbelu and Wu [15] presented the comparison of six dwell-point rules under dedicated and randomized storage policies using simulation. The authors compared the two formulations of Egbelu [13] and the four rules proposed by Bozer and White [9], and found that the solution from the minimum expected response time formulation performed well, as did the dwell-point strategy of Bozer and White [9] to always return to the input point. Hwang and Lim [16] showed that the formulations from Egbelu [9] could be transformed to facility location problems in order to reduce the computational time. These transformations reduced the required computational times by two orders of magnitude.

Peters et al. [17] proposed analytical models for determining the optimal dwell-point locations for the S/R machine. They used continuous rack approximation to provide analytical models of the dwell-point location problem. These models provide closed-form expressions for the dwell-point location in an AS/RS. A computational study of the effectiveness of the optimal dwell-point strategy is not provided in Peters et al. [17]. Chang and Egbelu [18,19] present formulations for pre-positioning of S/R machines to minimize the maximum system response time, and minimize the expected system response time for militarize AS/RS.

Van den Berg and Gademann [20] developed a simulation study of an AS/RS to examine a wide variety of control policies. The authors compared several storage location assignment policies. For the class-based storage policy, an algorithm was applied that evaluate the trade-off between storage space requirements and travel times. Also in this paper a new storage location policy was proposed that combines low storage space requirements with short mean travel times. Furthermore, the authors studied sequencing of storage and retrieval requests, whereby focused on the trade-off between efficient travel of the S/R machines and response time performance. Park [21] developed an optimal policy for automated storage/retrieval systems with uniformly distributed rack. A closed-form solution for the optimal dwell point was presented in terms of the probability of the next transaction demand type: storage or retrieval in a non-square-in-time rack. Various return paths to the dwell point were introduced in this research for the efficient operation of the S/R machine.

Thus, based on examination of the literature, although many dwell-point strategies have been suggested, and an optimal strategy defined, there does not appear to exist a computational study that illustrates the benefits of using the optimal dwell-point over the more simple rules suggested by Bozer and White [9]. Moreover, for AS/RSs with high system utilizations, it is not clear what opportunity exists in a practical sense to take advantage of the dwell-point strategies since the S/R machine will not be idle very often.

Literature in the area of travel-time models for S/R machines

Development of expected travel-time (i.e., average travel time) models for S/R machines is another area of research. A comparative study based on expected travel time of S/R machines for randomized and dedicated storage policies has been presented by Hausman et al. [11]. The rack configuration has been assumed to be square in time (i.e., horizontal maximum travel time is equal to vertical maximum travel time) with single and dual command cycles. An extension of Hausman et al. [11] has been proposed by Graves et al. [8]. An analytical and empirical result for various combinations of alternative storage assignment rules and scheduling policies was published in this paper. Each alternative was compared on the basis of the expected travel time of the S/R machine.Based on a continuous rack approximation approach, Bozer and White [9] present expressions for the

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expected cycle times of an AS/RS performing single and dual command cycles. Foley and Frazelle [22] derive the distribution of the dual command cycle time for a square-in-time rack under randomized storage and use it to determine the throughput of a Miniload AS/RS. A rack is square-in-time if the time to travel along the entire rack is the same in horizontal and vertical direction.

Hwang and Lee [23] presented travel-time models which include constant acceleration and deceleration rates with a maximum-velocity restriction. Chang et al. [24] proposed travel time models that consider various travel speeds with known acceleration and deceleration rates. Chang and Wen [25] extended the work presented by Chang et al. [24] by investigating the rack configuration problem. All these studies are basically focused on unitload AS/RS. Hu et al. [5] presented *split-platform AS/RS* (*SP-AS/RS)* to handle extra heavy loads such as sea container cargo. A reliable travel time model for this system was presented under stay dwell point policy.

CALCULATIONS FOR TRAVEL TIME MODELS

The travel time will be analyzed by using continuous models that considerably reduce the difficulty of the subsequent analysis [5, 9].

Assumptions and points

The following assumptions are made through this paper:
1. The rack is considered as a continuous rectangular pick face;
2. Platforms operate on single command basis;
3. Randomized storage is used which means that any point within the pick face is equally likely to be selected for storage or retrieval;
4. The specification of the rack is known;
5. In all of calculations there is no consideration to the any transfer time of the load due to its constant and little value (this refers to the time needed to transfer a load between VP and HP, between HP and an AS/RS cell or between the I/O station and the VP);
6. There is no acceleration in speed of VP or HP and it is assumed that empty and loaded VP and HP have same speed.

It also is assumed that during each operation, there is no prior information of subsequent job, so there are no simultaneously movements of VP or HP for next operation. Base on Hu et al. [5]; note that due to be free from the shape and the speed of any system, *b* is introduced as *shape factor,* which is

$$b = \frac{\textit{Maximum time for VP to go to the farthest point of handover station}}{\textit{Maximum time for HP to go to the farthest point of each tier}}$$

If HL is the length of the rack and VL is the height of rack, and *hv* and *vv* are the speeds of HP and VP then T_h= HL/*hv* and T_v= VL/*vv* and finally *b*= T_v/T_h. With all these symbols the rack is normalized as a rectangular pick face with length of 1 and height of b in term of time. Consider that these models give a number which is a ratio of T_h, so after calculations from the model it's necessary to multiply this ratio in T_h.

Review on stay dwell point policy

Hu et al. [5] developed a reliable travel time model for SP-AS/RS under stay dwell point policy. The expected travel time *E[T]* of this model is $E[T]=P(s)E[T_s]+P(r)E[T_r]$, where *T* denotes the cycle time for the S/R mechanism to complete an operation. T_s indicates the time spent if the current job is storage, while T_r is the time spend for a retrieval operation. Thus, it is clear that $E[T]$ denotes the expected travel time for one operation, $E[T_s]$ gives the expected travel time if the current job is storage and $P(s)$ is the probability for the current job to be storage operation. $E[T_r]$ and $P(r)$ are similarly defined for the case of retrieval. By definition, $P(r) = 1-P(s)$. Also assume that the ratio for storage operations is α in an arbitrary finite job sequence. In the case of infinite sequence of jobs, when the value of α is 1/2 the *E[T]* can be expressed as,

$$E[T]=\begin{cases}\frac{7}{640}b^3+\frac{21}{192}b^2+\frac{7}{16}b+\frac{5}{6} & ,0<b\le\frac{1}{2}\\ \frac{7}{640}b^3+\frac{5}{192}b^2+\frac{29}{48}b+\frac{1}{24b}-\frac{1}{192b^2}-\frac{17}{24} & ,\frac{1}{2}\le b\le 1\\ -\frac{7}{1920}b^3+\frac{3}{64}b^2+\frac{5}{8}b+\frac{5}{96b}-\frac{1}{960b^2}+\frac{2}{3} & ,1\le b\le 2\\ \frac{5}{6}b+\frac{41}{96b}+\frac{113}{960b^2}+\frac{1}{4} & ,b\ge 2\end{cases}$$

Analysis for return to middle policy

Let (*x,y*) denotes the target point of current job. The cycle time for a storage operation (figure 3.) starts from middle point of handover station (*b/2*) which VP goes to I/O station and the load is transferred to VP and then VP goes to the corresponding tier in handover station (*y*), simultaneously HP of related tier moves from the middle point of that tier (*1/2*) to the handover station and the load is transferred to HP. The HP in next movement carries the load to the destination cell (*x*) and after storage the load, returns to the middle point of tier (*1/2*), concurrently the VP moves to the middle point of handover station (*b/2*) and the storage operation completes after reaching VP and HP to their dwell points.

For a retrieval operation the cycle time is just the reverse of storage operation, but the travel time of retrieval operation is same as storage operation so,

$$T=\max(b/2+y,\ 1/2)+\max(|x-1/2|+x,\ |y-b/2|)$$

To achieve a good model for this travel time the area of each rack is divided into 4 separated zones which illustrated in figure 3. Consider that in each zone, the T is calculated independently and with the specification of that zone. Also consider that each zone is a separate rack. Finally these results are added to each other through $E[T]=0.25\sum_{i=1}^{4}E[T_i]$ and final result is achieved. Consider that because probability of selecting each zone is equal to other zones so the probability of selecting each zone is 0.25.

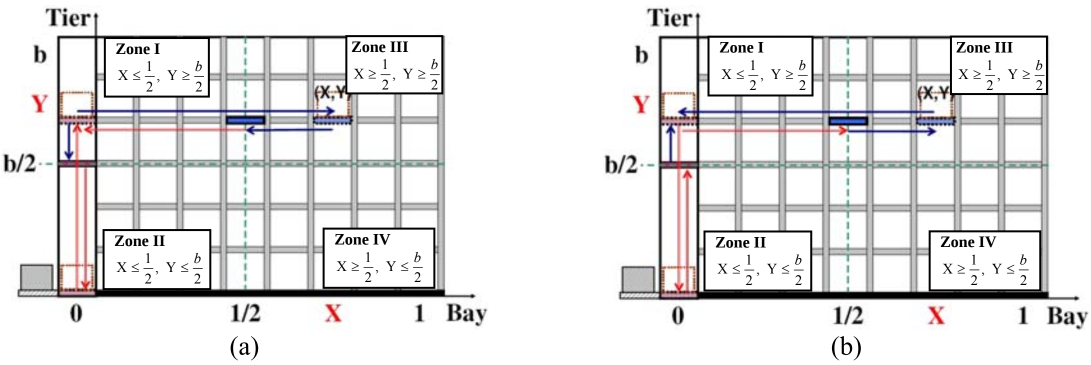

Figure 3: Illustration of storage (a) and retrieval (b) operations in return to middle policy and 4 zones for their calculations

Calculations for first zone

For first zone x≤1/2 and y≥b/2 so,

$T_1=\max(b/2+y,\ 1/2)+\max(1/2,y-b/2)$ and $E[T_1]=E[\max(b/2+y,\ 1/2)]+E[\max(1/2,y-b/2)]$

In the first area b/2≤y≤b and randomized storage and retrieval is used so,

$$F_y(v)=\begin{cases}0 & ,v\le\frac{b}{2}\\ \frac{2v}{b}-1 & ,\frac{b}{2}\le v\le b\\ 1 & ,v\ge b\end{cases}\quad\text{and}\quad f_y(v)=\begin{cases}\frac{2}{b} & ,\frac{b}{2}\le v\le b\\ 0 & ,otherwise\end{cases}$$

Thus in this area $E[y]=\int_{b/2}^{b}2v/b\ dv=3b/4$

For max (b/2+y, 1/2), the limitation of y in this zone is $b/2 \le y \le b$ so $b \le y+b/2 \le 3b/2$ (1), now if the max limitation of (1) is less than 1/2, it can be resulted that always $b/2+y \le 1/2$ thus,

If $3b/2 \le 1/2$ then $b/2+y \le 1/2$, it means that *if $b \le 1/3$ then $b/2+y \le 1/2$* so in this area*, max (b/2+y, 1/2) is 1/2* and if the minimum limitation of (1) is greater than 1/2, consider that always $b/2+y \ge 1/2$ thus,

If $b \ge 1/2$ then $b/2+y \ge 1/2$ so for this area *max (b/2+y, 1/2) is b/2+y.*

For the area $1/3 \le y \le 1/2$ consider that, for $y+b/2 \le 1/2$, the limitation of y in this zone is $y \le \frac{1-b}{2}$ so,

for $b/2 \le y \le \frac{1-b}{2}$, then $y+b/2 \le 1/2$ * and for $\frac{1-b}{2} \le y \le b$ then $y+b/2 \ge 1/2$ ^{2*}

so in this area $E[T]=P^{*} \times E[1/2]+P^{2*} \times E[y+b/2]$ with;

$$P^{*} = \frac{\frac{1-b}{2}-\frac{b}{2}}{b-\frac{b}{2}} = \frac{1-2b}{b} \qquad \text{and} \qquad P^{2*} = 1-\frac{1-2b}{b} = \frac{3b-1}{b}$$

For max (1/2, y-b/2) with same analysis $b/2 \le y \le b$ then $0 \le y-b/2 \le b/2$, (2), now if the maximum limitation of (2) is less than 1/2 it can be resulted that always $y-b/2 \le 1/2$ thus, *if $b \le 1$ then always $y-b/2 \le 1/2$* and if $b \ge 1$ consider that,

for $y-b/2 \le 1/2$, then $y \le 1+b/2$ thus for $b/2 \le y \le 1+b/2$, then $y-b/2 \le 1/2$ ^{3*} and for $1+b/2 \le y \le b$ then $y-b/2 \ge 1/2$ ^{4*}

so in this area $E[T]=P^{3*} \times E[1/2]+P^{4*} \times E[y-b/2]$, with:

$$P^{3*} = \frac{\frac{1+b}{2}-\frac{b}{2}}{b-\frac{b}{2}} = \frac{1}{b} \qquad \text{and} \qquad P^{4*} = 1-\frac{1}{b} = \frac{b-1}{b}$$

Now considering the above relations, the expected value of travel time for the first zone is,

$$E[\max(b/2+y,\ 1/2)] = \begin{cases} E[\frac{1}{2}] & ,b \le \frac{1}{3} \\ \frac{1-2b}{b}E[\frac{1}{2}]+\frac{3b-1}{b}(E[y+\frac{b}{2}]) & ,\frac{1}{3} \le b \le \frac{1}{2} \\ E[y+\frac{b}{2}] & ,b \ge \frac{1}{2} \end{cases} = \begin{cases} \frac{1}{2} & ,b \le \frac{1}{3} \\ \frac{1-2b}{2b}+\frac{5(3b-1)}{4} & ,\frac{1}{3} \le b \le \frac{1}{2} \\ \frac{5b}{4} & ,b \ge \frac{1}{2} \end{cases}$$

$$\text{and } E[\max(1/2,y-b/2)] = \begin{cases} E[\frac{1}{2}] & ,b \le 1 \\ \frac{1}{b}E[\frac{1}{2}]+\frac{b-1}{b}E[y-\frac{b}{2}] & ,b \ge 1 \end{cases} = \begin{cases} \frac{1}{2} & ,b \le 1 \\ \frac{1}{2b}+\frac{b-1}{4} & ,b \ge 1 \end{cases}$$

Hence,

$$E[T_1] = \max(b/2+y,\ 1/2)+\max(1/2,y-b/2)] = \begin{cases} 1 & ,b \le \frac{1}{3} \\ \frac{1-2b}{2b}+\frac{5(3b-1)}{4}+\frac{1}{2} & ,\frac{1}{3} \le b \le \frac{1}{2} \\ \frac{5b}{4}+\frac{1}{2} & ,\frac{1}{2} \le b \le 1 \\ \frac{5b}{4}+\frac{1}{2b}+\frac{b-1}{4} & ,b \ge 1 \end{cases} \qquad (3)$$

Calculations for second zone

The limitations of second zone are x ≤1/2 and y ≤b/2 so,

T_2=max(b/2+y, 1/2)+max(1/2,b/2-y) and *E[T_2]= E[max (b/2+y, 1/2+ E[max (1/2,b/2-y)]*

In this zone the limitations are 0≤y≤b/2 so,

$$F_y(v)=\begin{cases}0 & ,v\le 0\\ \frac{2v}{b} & ,0\le v\le \frac{b}{2}\\ 1 & ,v\ge \frac{b}{2}\end{cases} \quad \text{and} \quad f_y(v)=\begin{cases}\frac{2}{b} & ,0\le v\le \frac{b}{2}\\ 0 & ,otherwise\end{cases}$$

Thus in this area $E[y] = \int_0^{b/2} 2v/b \; dv = b/4$.

Now in this area with same analysis for the first zone:

for $b \le 1/2$, *max (b/2+y, 1/2)* is 1/2 and for $b\ge 1$ the *max (b/2+y,1/2)* is *b/2+y*
for 1/2≤y≤1 should divide it into two areas, first area, $0\le y\le \frac{1-b}{2}$, that max(b/2+y,1/2) is 1/2 and its probability is $\frac{1-b}{b}$ and second area, $\frac{1-b}{2}\le y\le b/2$ that max(b/2+y,1/2) is b/2+y and its probability is $\frac{2b-1}{b}$ so,

$$\text{E[max(b/2+y,1/2]} = \begin{cases} E[\frac{1}{2}] & ,b\le\frac{1}{2}\\ \frac{1-b}{b}E[\frac{1}{2}]+\frac{2b-1}{b}E[y+\frac{b}{2}] & ,\frac{1}{2}\le b\le 1\\ E[y+\frac{b}{2}] & ,b\ge 1\end{cases} = \begin{cases}\frac{1}{2} & ,b\le\frac{1}{2}\\ \frac{1-b}{2b}+\frac{3(2b-1)}{4} & ,\frac{1}{2}\le b\le 1\\ \frac{3b}{4} & ,b\ge 1\end{cases}$$

and for max(b/2-y,1/2) if b≤1 then max(b/2-y,1/2) is 1/2 and if b≥1 should divide it into two areas, first area, $0\le y\le \frac{b-1}{2}$, that max(b/2-y,1/2) is (b/2-y) and its probability is $\frac{b-1}{b}$, and second area, $\frac{b-1}{2}\le y\le b/2$, that max(b/2-y, 1/2) is 1/2 and its probability is 1/b so,

$$\text{E[max(b/2+y,1/2]} = \begin{cases} E[\frac{1}{2}] & ,b\le 1\\ \frac{1}{b}E[\frac{1}{2}]+\frac{b-1}{b}E[\frac{b}{2}-y] & ,b\ge 1\end{cases} = \begin{cases}\frac{1}{2} & ,b\le 1\\ \frac{1}{2b}+\frac{b-1}{4} & ,b\ge 1\end{cases}$$

Hence,

$$\text{E[T}_2\text{]= E[max(b/2+y,1/2)+ max(1/2,b/2-y)]} = \begin{cases}1 & ,b\le\frac{1}{2}\\ \frac{1-b}{2b}+\frac{3(2b-1)}{4}+\frac{1}{2} & ,\frac{1}{2}\le b\le 1\\ b+\frac{1}{2b}-\frac{1}{4} & ,b\ge 1\end{cases} \tag{4}$$

Calculations for third zone

In the third zone $x \geq 1/2$ and $y \geq b/2$ so,

$T_3 = max(b/2+y,1/2)+max(2x-1/2,y-b/2)$ and $E[T_3] = E[max(b/2+y,1/2)]+E[max(2x-1/2,y-b/2)]$

In this zone:

$$f_x(v)=\begin{cases} 2 & ,\frac{1}{2} \le v \le 1 \\ \\ 0 & ,otherwise \end{cases} \quad \text{(a.1)} \qquad f_y(v)=\begin{cases} \frac{2}{b} & ,\frac{b}{2} \le v \le b \\ \\ 0 & ,otherwise \end{cases} \quad \text{(a.2)}$$

now consider that calculations for E[max(b/2+y,1/2)] is same as calculations in first zone (T_1) so,

$$E[max(b/2+y,1/2)] = \begin{cases} \frac{1}{2} & ,b \le \frac{1}{3} \\ \frac{1-2b}{2b}+\frac{5(3b-1)}{4} & ,\frac{1}{3} \le b \le \frac{1}{2} \\ \frac{5b}{4} & ,b \ge \frac{1}{2} \end{cases} \quad \text{(a.3)}$$

and for E[max(2x-1/2,y-b/2)] let Z=max(2x-1/2,y-b/2) so because (2x-1/2) and (y-b/2) are independent of our analysis of the new mechanism, the probability function of Z can be expressed as

$$F(Z \le v)=P(Z \le v)=P(2x-1/2 \le v)P(y-b/2 \le v) \quad \text{(a.4)}$$

now if M denotes 2x-1/2, then P(2x-1/2≤v) = P(x≤$\frac{v+1/2}{2}$), based on (a.1), consider $F_M(v) = \int_{1/2}^{\frac{v+1/2}{2}} 2 \, dv$ so,

$$F_M(v)=\begin{cases} 0 & ,v \le \frac{1}{2} \\ v-\frac{1}{2} & ,\frac{1}{2} \le v \le \frac{3}{2} \\ 1 & ,v \ge \frac{3}{2} \end{cases} \quad \text{(a.5)}$$

and if N denotes y-b/2, then P(y-b/2≤v)= P(y≤v+b/2), based on (a.2), consider $F_N(v)=\int_{b/2}^{v+b/2} 2/b \, dv$ so,

$$F_N(v)=\begin{cases} 0 & ,v \le 0 \\ \frac{2v}{b} & ,0 \le v \le \frac{b}{2} \\ 1 & ,v \ge \frac{b}{2} \end{cases} \quad \text{(a.6)}$$

Now, by substituting Eqs. (a.5) and (a.6) into (a.4) then:

When b≤1

$$F_Z(v) = \begin{cases} 0 & ,v \le \frac{1}{2} \\ v-\frac{1}{2} & ,\frac{1}{2} \le v \le \frac{3}{2} \\ 1 & ,v \ge \frac{3}{2} \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} 1 & ,\frac{1}{2} \le v \le \frac{3}{2} \\ \\ 0 & ,otherwise \end{cases} \quad \text{(a.7)}$$

when 1≤b≤3

$$F_Z(v) = \begin{cases} 0 & ,v\le\frac{1}{2} \\ \frac{2v^2-v}{b} & ,\frac{1}{2}\le v\le\frac{b}{2} \\ v-\frac{1}{2} & ,\frac{b}{2}\le v\le\frac{3}{2} \\ 1 & ,v\ge\frac{3}{2} \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \frac{4v-1}{b} & ,\frac{1}{2}\le v\le\frac{b}{2} \\ 1 & ,\frac{b}{2}\le v\le\frac{3}{2} \\ 0 & ,otherwise \end{cases} \tag{a.8}$$

when b≥3

$$F_Z(v) = \begin{cases} 0 & ,v\le\frac{1}{2} \\ \frac{2v^2-v}{b} & ,\frac{1}{2}\le v\le\frac{3}{2} \\ \frac{2v}{b} & ,\frac{3}{2}\le v\le\frac{b}{2} \\ 1 & ,v\ge\frac{b}{2} \end{cases} \quad \text{and} \quad f_Z(v) = \begin{cases} \frac{4v-1}{b} & ,\frac{1}{2}\le v\le\frac{3}{2} \\ 1 & ,\frac{3}{2}\le v\le\frac{b}{2} \\ 0 & ,otherwise \end{cases} \tag{a.9}$$

Hence, based on Eqs. (a.7) – (a.9) the expected value of Z is

$$E[Z] = \int_v v f_z(v)\ dv = \begin{cases} 1 & ,b\le 1 \\ \frac{4b^2-3b}{24}+\frac{9-b^2}{8}-\frac{1}{24b} & ,1\le b\le 3 \\ \frac{10}{3b}+\frac{b^2-9}{4b} & ,b\ge 3 \end{cases} \tag{a.10}$$

And finally adding (a.3) to (a.10) then,

$$E[T_3] = \begin{cases} \frac{3}{2} & ,b\le\frac{1}{3} \\ \frac{1-2b}{2b}+\frac{5(3b-1)}{4}+1 & ,\frac{1}{3}\le b\le\frac{1}{2} \\ \frac{5b}{4}+1 & ,\frac{1}{2}\le b\le 1 \\ \frac{4b^2-3b}{24}+\frac{9-b^2}{8}-\frac{1}{24b}+\frac{5b}{4} & ,1\le b\le 3 \\ \frac{10}{3b}+\frac{b^2-9}{4b}+\frac{5b}{4} & ,b\ge 3 \end{cases} \tag{5}$$

Calculations for forth zone

In the forth zone the limitations are x≥1/2 and y≤b/2 so,

T_4= max(b/2+y,1/2) + max(2x-1/2,b/2-y) and *E[T_4]=E[max(b/2+y,1/2)]+E[max(2x-1/2,b/2-y)]*

$$f_x(v)= \begin{cases} 2 & ,\frac{1}{2}\le v\le 1 \\ 0 & ,otherwise \end{cases} \tag{b.1}$$

$$f_y(v)= \begin{cases} \frac{2}{b} & ,0\le v\le\frac{b}{2} \\ 0 & ,otherwise \end{cases} \tag{b.2}$$

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now consider that E[max(b/2+y,1/2)] is same as calculations in second zone (T_2) so,

$$E[\max(b/2+y,1/2)] = \begin{cases} \frac{1}{2} & ,b \le \frac{1}{2} \\ \frac{1-b}{2b} + \frac{3(2b-1)}{4} & ,\frac{1}{2} \le b \le 1 \\ \frac{3b}{4} & ,b \ge 1 \end{cases} \quad \text{(b.3)}$$

and for E[max(2x-1/2,b/2-y)] let Z = max(2x-1/2,b/2-y) so because 2x-1/2 and b/2-y are independent of the analysis of new mechanism, the probability function of Z can be expressed as

F(Z≤v)=P(Z≤v)=P(2x-1/2≤v)P(b/2-y≤v)

now if M denotes 2x-1/2, then P(2x-1/2≤v) = P(x≤$\frac{v+1/2}{2}$), based on (b.1) consider, $F_M(v) = \int_{1/2}^{\frac{v+1/2}{2}} 2 \; dv$ so,

$$F_M(v) = \begin{cases} 0 & ,v \le \frac{1}{2} \\ v - \frac{1}{2} & ,\frac{1}{2} \le v \le \frac{3}{2} \\ 1 & ,v \ge \frac{3}{2} \end{cases}$$

and if N denotes b/2-y, then P(b/2-y≤v)=P(y≥b/2-v)=1-P(y≤b/2-v), based on (b.2), $F_N(v)=1-\int_0^{b/2-v} 2/b \; dv$ so,

$$F_N(v) = \begin{cases} 0 & ,v \le 0 \\ \frac{2v}{b} & ,0 \le v \le \frac{b}{2} \\ 1 & ,v \ge \frac{b}{2} \end{cases}$$

The calculation for $F_N(v)$ is quite similar to a.6 to a.9, so the expected value of Z is:

$$E[Z] = \int_v v f_z(v) \; dv = \begin{cases} 1 & ,0 \le b \le 1 \\ \frac{4b^2-3b}{24} + \frac{9-b^2}{8} - \frac{1}{24b} & ,1 \le b \le 3 \\ \frac{10}{3b} + \frac{b^2-9}{4b} & ,b \ge 3 \end{cases} \quad \text{(b.4)}$$

And finally adding (b.3) to (b.4) then,

$$E[T_4] = \begin{cases} \frac{3}{2} & ,b \le \frac{1}{2} \\ \frac{1-b}{2b} + \frac{3(2b-1)}{4} + 1 & ,\frac{1}{2} \le b \le 1 \\ \frac{4b^2-3b}{24} + \frac{9-b^2}{8} + \frac{3b}{4} - \frac{1}{24b} & ,1 \le b \le 3 \\ \frac{10}{3b} + \frac{b^2-9}{4b} + \frac{3b}{4} & ,b \ge 3 \end{cases} \quad (6)$$

Now consider that E[T]= $0.25\sum_{i=1}^{4}E[T_i]$ so the final model for this policy based on (3) – (6)is:

$$E[T]=\begin{cases}\frac{5}{4} & ,b\le\frac{1}{3}\\ \frac{1-2b}{4b}+\frac{15b}{8}+\frac{3}{8} & ,\frac{1}{3}\le b\le\frac{1}{2}\\ \frac{11b}{8}+\frac{1-b}{4b}+\frac{3}{8} & ,\frac{1}{2}\le b\le 1\\ \frac{b^2}{48}+\frac{17b}{16}+\frac{11}{48b}+\frac{7}{16} & ,1\le b\le 3\\ \frac{10b}{8}+\frac{19}{24b}-\frac{1}{8} & ,b\ge 3\end{cases}$$

Analysis for return to start policy

Now consider another model that after each operation HPs return to hand-over stations and VP returns to I/O station so the equation for T change to (Appendix B),

$$T= y+ \max(2x,y) \quad\text{and}\quad E[T] = E[y] + E[\max(2x,y)] \tag{c.1}$$

This policy was considered in Hu et al. [6] but used a discrete model for their calculations, in this article the authors introduce a new continuous model for it and finally validate it by simulation. Consider that in this policy, like the pervious one the cycle time for storage and retrieval operations are same as each other. As randomized storage is used, the probability distribution function and probability density function of x are as follows:

$$F_x(v)=\begin{cases}v & ,0\le v\le 1\\ 1 & ,v\ge 1\end{cases} \quad\text{and}\quad f_x(v)=\begin{cases}1 & ,0\le v\le 1\\ 0 & ,otherwise\end{cases} \tag{c.2}$$

and $E[x]=\int_0^1 v\times f_x(v)dv=\frac{1}{2}$ (c.3)

Meanwhile, the probability distribution function and the probability density function of y are:

$$F_y(v)=\begin{cases}\frac{v}{b} & ,0\le v\le b\\ 1 & ,v\ge b\end{cases} \quad\text{and}\quad f_y(v)=\begin{cases}\frac{1}{b} & ,0\le v\le b\\ 0 & ,otherwise\end{cases} \tag{c.4}$$

and $E[y]=\int_0^b v\times f_y(v)dv=\frac{b}{2}$ (c.5)

for E[max(2x,y)], let Z = max(2x,y) then $F_Z(V)=P(Z\le v)$, and because $2x$ and y are independent from our analysis of the new mechanism so,

$$P(Z\le v)=P(2x\le v)\times P(y\le v), \tag{c.6}$$

Now consider that, P(2x≤v)=P(x≤v/2) so considering (c.2), then, $F_x(v)=\int_0^{v/2}1\,dv$, so

$$F_x(v)=\begin{cases}0 & ,v\le 0\\ \frac{v}{2} & ,0\le v\le 2\\ 1 & ,v\ge 2\end{cases} \tag{c.7}$$

and for P(y≤v) considering (c.4), then:

$$F_y(v)=\begin{cases}0 & ,v\le 0\\ \frac{v}{b} & ,0\le v\le b\\ 1 & ,v\ge b\end{cases} \tag{c.8}$$

now by substituting (c.7) and (c.8) in (c.6), then:

when b≤2

$$F_Z(v)=\begin{cases}0 & ,v\le 0\\ \frac{v^2}{2b} & ,0\le v\le b\\ \frac{v}{2} & ,b\le v\le 2\\ 1 & ,v\ge 2\end{cases} \quad \text{and} \quad f_Z(v)=\begin{cases}\frac{v}{b} & ,0\le v\le b\\ \frac{1}{2} & ,b\le v\le 2\\ 0 & ,otherwise\end{cases} \tag{c.9}$$

when b≥2

$$F_Z(v)=\begin{cases}0 & ,v\le 0\\ \frac{v^2}{2b} & ,0\le v\le 2\\ \frac{v}{2} & ,2\le v\le b\\ 1 & ,v\ge b\end{cases} \quad \text{and} \quad f_Z(v)=\begin{cases}\frac{v}{b} & ,0\le v\le 2\\ \frac{1}{2} & ,2\le v\le b\\ 0 & ,otherwise\end{cases} \tag{c.10}$$

Now based on equations (c.9) and (c.10) then: $E[Z]=\int_v v f_z(v)\ dv$ so,

$$E[Z]=\begin{cases}\frac{b^2}{12}+1 & ,b\le 2\\ \frac{8}{3b}+\frac{b^2-4}{2b} & ,b\ge 2\end{cases} \tag{c.11}$$

Finally, substituting (c.5) and (c.11) into (c.1), then E[T] in this policy is:

$$E[T]=\begin{cases}\frac{b^2}{12}+\frac{b}{2}+1 & ,b\le 2\\ \frac{8}{3b}+\frac{b^2-4}{2b}+\frac{b}{2} & ,b\ge 2\end{cases}$$

VALIDATION OF TRAVEL TIME MODELS

To evaluate the continuous models for their accuracy, the results obtained from the model are compared with those from the computer simulations. This study presents experiments or simulation models using MS Excel®.

Under these policies, the ratio of storage operations vs. retrieval operations, and consequently the sequence of operations are not important. The simulation contains the randomized number generation for *x* and *y* to choose a new destination for new operation. Then the exact equation of T is used for this randomized destination. After obtaining these results the average of all simulated results is calculated, and this average must be approximately near to the results of continuous model.

In order to make these results comparable to each other, same layout and specification is used for experiments.

These specifications are:
1. The number of total cells in each rack is 288;
2. The height and the width of each cell is 4.5m;
3. The VP travels at 1 m/s and the speed of HP is 2 m/s.

A sequence of 100000 jobs is used in the simulation and the real travel time equation is calculated then for better comparison of model results and simulation results the following equations is used.

$$\% \text{ Deviation} = \frac{\text{Model results- simulation results}}{\text{Simulation results}} \times 100$$

Considering Tables 1 and 2, it can be observed that the maximum % deviation is less than 3%, and the continuous models display a satisfactory performance with the largest percentage deviation being 2.67%. This shows that the continuous models perform quite well and they are entirely suitable for further researches.

Table1: Simulation results for return to middle policy

No. of tiers	No. of bays	Cells in rack	Shape factor, b	Simulation results	Model results	% Deviation
1	288	288	0.01	811.14	810	0.14
9	32	288	0.56	97.51	96.69	0.85
12	24	288	1.00	93.48	94.50	1.09
17	17	288	2.00	107.14	105.59	1.45
24	12	288	4.00	137.98	136.97	0.73
48	6	288	16.00	268.29	268.98	0.26
96	3	288	64.00	537.94	539.24	0.23
288	1	288	576.00	1617.02	1619.72	0.14

Table2: Simulation results for return to start policy

No. of tiers	No. of bays	Cells in rack	Shape factor, b	Simulation results	Model results	% Deviation
1	288	288	0.01	649.28	650.25	0.15
9	32	288	0.56	93.77	94.15	0.40
12	24	288	1.00	84.48	85.50	1.21
17	17	288	2.00	86.93	89.25	2.67
24	12	288	4.00	109.73	112.50	2.52
48	6	288	16.00	213.32	216.56	1.52
96	3	288	64.00	428.86	432.07	0.75
288	1	288	576.00	1294.55	1296	0.11

Figure 4 depicts the % deviation between the continuous models results and simulations results, for evaluating the accuracy of continuous models. As illustrated in this graph, The Peak points of the curves of % deviation, the locations where the curves are highest, occur when the racks become square (when the number of tiers and bays are same or close to each other, obviously it occurs around the shape factor 2). So from these graphs it can be observed that the performance of continuous models improve as the racks become non-square.

COMPARISON OF THREE DWELL POINT POLICIES FOR AS/RS

From table 3 and figure 5, it can be observed that under stay dwell policy (results for this policy are from Hu et al. [5]) expected operation time has its minimum value in compare with other policies, for varying shape factor. It can be concluded that stay dwell point policy outperforms other policies for a stable system where the fraction of storage and retrieval are identical. The optimal rack design will largely depends on the characteristics of the demand in our application. From a long term point of view the container yard has balanced work-flow which suggests that policy A be more preferable.

Table 3: Comparison of simulation results of 3 policies

No. of tiers	No. of bays	Cells in rack	Shape factor, b	Policy A	Policy B	Policy C
1	288	288	0.01	541.97	810	650.25
9	32	288	0.56	80.35	96.69	94.15
12	24	288	1.00	74.84	94.50	85.50
17	17	288	2.00	80.36	105.59	89.25
24	12	288	4.00	99.43	136.97	112.50
48	6	288	16.00	183.73	268.98	216.56
96	3	288	64.00	361.73	539.24	432.07
288	1	288	576.00	1080.56	1619.72	1296

Policies A, B, and C represent stay, return to middle and return to start dwell point policies
Results for policy A are from Hu et al. [5]

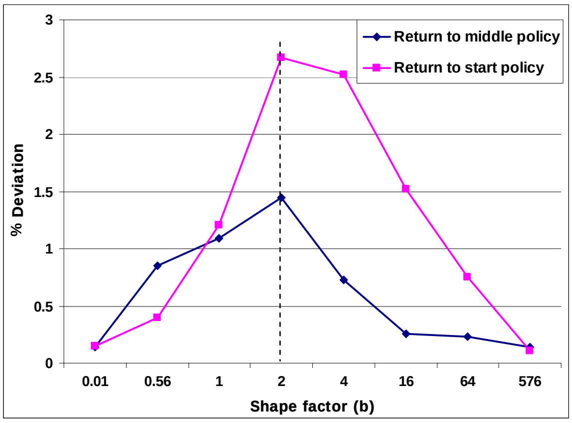


***Figure 4:** % deviation for travel time models*

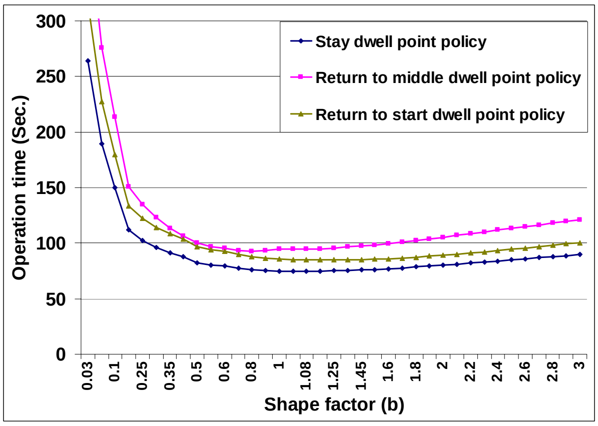
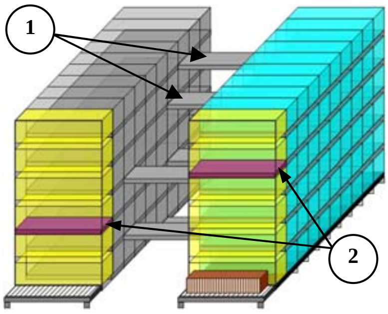


***Figure 5:** Influence of shape factor on operation time in varying policies*

CONCLUSION

Split-platform AS/RS, or *SP-AS/RS* for short is a new automated storage and retrieval system which is suitable to handle extra heavy loads such as loads above 20 tons at high speed. Two continuous travel-time models for the SP-AS/RS under the return to middle and return to start dwell point policies are developed. The models have been validated by the computer simulation. The results for these policies are compared with results for stay dwell point policy, which has been calculated in previous literature. The results show that in a varying shape factor the stay dwell point policy outperforms the new calculated policies. The results show that the lowest travel time occurs when shape factor is around 1, which means in constructing SP-AS/RS, one needs to consider both dimensions and the speed of S/R machines. Using travel time models, the structure of SP-AS/RS can be designed easily. In the next paper of this series, authors will present a new optimal configuration alternative for SP-AS/RS in order to reduce average handling time. The preliminary research has shown that, by using this new configuration alternative, the average handling time for a range of shapes can be greatly reduced.

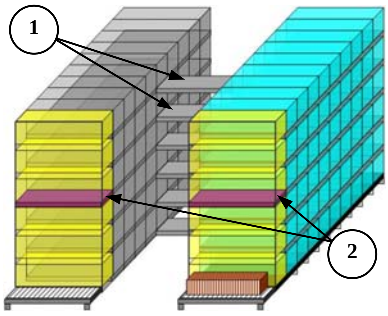
Appendix A



1. The position of HPs depends on the last operation in each tier

2. The position of VPs depends on the previous operation in each rack

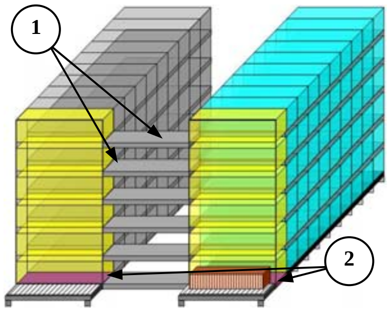
Figure A1: Stay dwell point policy



1. The HP returns to middle of tier upon finishing a job

2. The VP returns to middle of handover station upon finishing a job

Figure A2: Return to middle dwell point policy

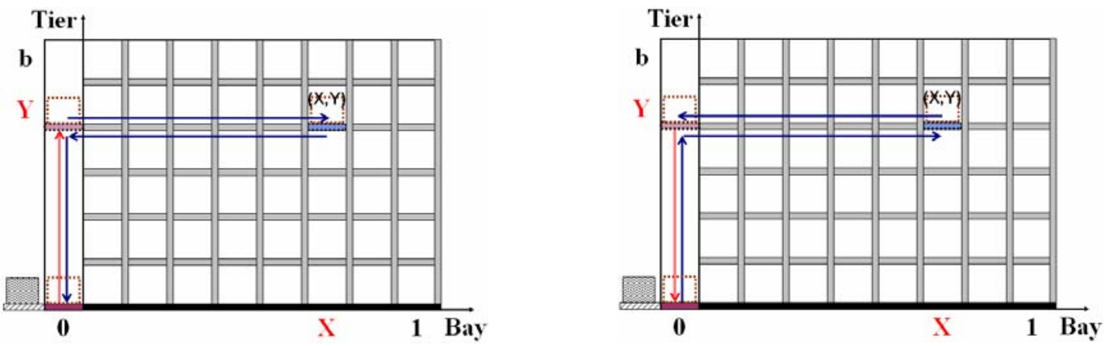


1. The HP returns to handover station upon finishing a job

2. The VP returns to I/O station upon finishing a job

Figure A3: Return to start dwell point policy

Appendix B



(a) **(b)**

Figure B1: Illustration of storage (a) and retrieval (b) operations in return to start policy

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