

ENERGY PREDICTION MODEL FOR DISK PLOW COMBINED WITH A ROTARY BLADE IN WET CLAY SOIL

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ABSTRACT

A mathematical model was developed to predict the energy requirement for the combined effect of a disk plow and a rotary blade in clay soil suitable for wet rice cultivation in Malaysia. The developed equations can be used to predict the energy requirements for disk and rotary blade using mathCAD PLUS 6.0 software. The validity of the model was checked via experiments conducted in an indoor soil tank with the usage of an octagonal ring transducer for the disk and a torque meter for the rotary blade. The variations between experimental and predicted values range from -1.4% to 3.3%.

Key words: *Mathematical Model, Energy Requirement, Disk Plow, Rotary Blade, Clay Soil*

INTRODUCTION

Puddling is one of the important farm operations in paddy cultivation especially in wet clay soil in South-East Asia. Puddling is the process of churning the soil and water in a flooded field so as to form homogenous mixture such that it facilitates transplanting of seedlings. To achieve the puddled conditions, disk plows, country plows, harrows and rotary tillers are widely used. Since the behavior of wet clay soil under the action of tillage is still not fully understood, the design of wet clay soil working implement has to rely upon empirical equations or by trial and error.

In the past, various mathematical models have been developed by many researchers to predict the forces acting on disk plow and rotary blade separately [1-7]. The physical size and shape of a disc, its setting relative to the direction of motion through the soil and the properties of the soil itself determine its performance characteristics. An understanding of the interaction between disc and soil would allow the prediction of disc performance from known soil and disc parameters. A disc could then be set to operate in a particular soil at a predetermined depth or with minimum draught force. In this regards, Abo El Ees and Wills [4] studied the geometrical interaction between a vertical disc and the soil in terms of disc geometry, disc setting and working depth. They concluded that the radius of curvature of the disc sphere and the diameter of the circle formed by the disc edge relate directly to the geometry of the disc, whereas the disc angle and the depth of penetration define the attitude and position of the disc within the soil mass. In the measurement of draft on disk plow, it increases in linear fashion with increase of tilt angles, disk angle (from 45 to 50 deg) and the depth of cut [8]. Soehne [9] operated experimental rotary tiller blades in a soil bin to study various geometry factors (blade width, cutting angle, radius of curvature), speeds of operation, and cutting operations. Tsuchiya and Honami [6] studied blade sharpness, cutting characteristics of specific blade shape and arrangement of blades on the rotor using a model soil bin and half scale tiller blades. Gupta and Visvanathan [7] developed a mathematical model for predicting power requirement of a rotary tiller. In saturated lateritic sandy clay loam soil at 72.4% moisture content using L-shaped tines, they found that the power requirement of a rotary tiller consists of 0.34-0.59% for cutting soil slices, 30.5-72.4% for throwing out soil slices, 0.96-2.45% for overcoming soil-metal friction, 0.62-0.99% for soil-soil friction and 23.1-64.6% for idle power.

The past research work did not consider the combined effect of disk plow and rotary blades in wet clay soil. In this article we have attempted to predict the power requirement of combined disk-rotary blade by a mathematical model. Its validity was checked by conducting experiments in wet clay soil in an indoor soil tank.

MATHEMATICAL MODELS FOR ENERGY PREDICTION

Disk Plow

Disk plows are well adapted to plowing in extremely hard soil; for cutting, pulverizing, elevating, and inverting furrow slices in primary as well as in secondary tillage. The factors affecting forces on disk plow include type of soil, the bearing of disk, scraper type, tilt angle, disk angle, forward speed, depth, and width of cut [10 and 11].

Characteristic Soil Flow On Disk Surface Model

To obtain optimum functional design of disk plow, it is indispensable to study the overturning of furrow slice in wet soil tillage. This will be carried out to clarify characteristics of the overturning of furrow slice.

Disk Geometry And Disk Angle

A disk can be considered to be part of the shell of a hollow sphere. The different dimensions are shown in Figure 1.0

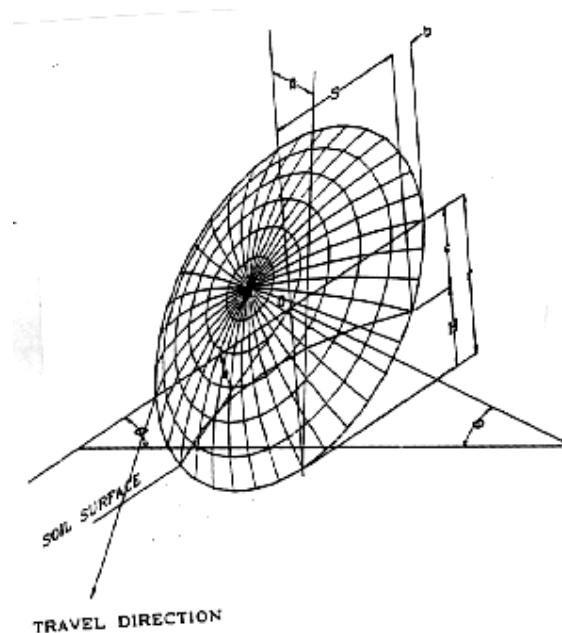


Figure 1.0 A disk plow shape and its angle [reference 4]

In Figure 1.0, r is the radius of the disk edge circle, H is disk penetration into soil, β is tilt angle, ϕ is disk angle, c is the vertical distance of soil surface from the center of diameter at the disk edge, S is the horizontal distance between soil surface and the horizontal, and θ is the angle between soil surface and the horizontal. From Figure 1.0, the following equations can be derived where

$$r = d/(2\cos\beta); c = r - H; S = \sqrt{(r^2 - c^2)} \text{ and } S = r\cos\theta$$

Rearranging the above equations gives,

$$\theta = a \cos \frac{\sqrt{2rH - H^2}}{r} \quad \dots(1)$$

Disk Spacing And Depth

The disk spacing and cut were associated with the disk angle. For a disk at its depth and angle, soil contact will take place only on the convex side of the disk, at the cutting edge of the disk, and the plane of the soil surface. If the disk angle is decreased or the working depth increased, soil contact will spread over the convex surface of the disk [12]. The relationship is shown graphically in Figure 2.0.

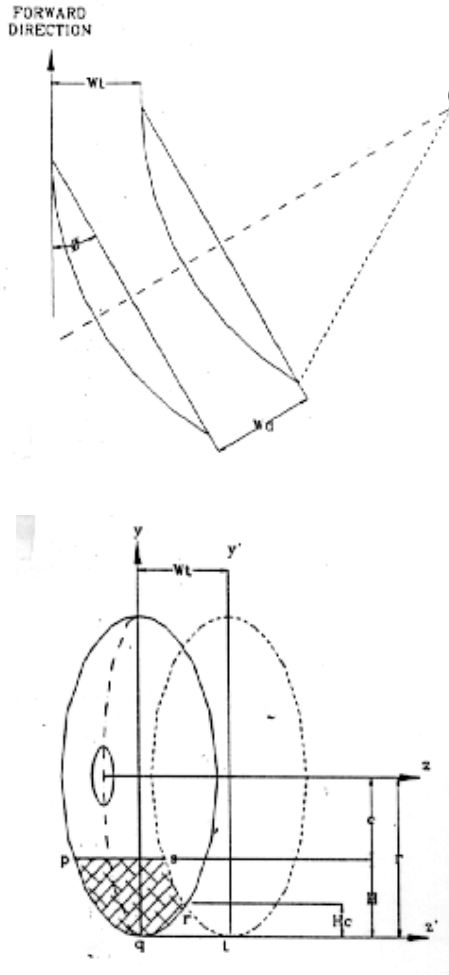


Figure 2.0 Relationship between disk spacing, tillage width and the apparent height of tillage residual.

From Figure 2.0, co-ordinates of y and z are: $y = -r \cos \theta$ and $z = -r \sin \phi \sin \theta$. From these two values, we can get, $y = -[\sqrt{\{(r \sin \phi)^2 - z^2\}}] / \sin \phi$

From z' axis, value of y will be, $y = r - [\sqrt{\{(r \sin \phi)^2 - z^2\}}] / \sin \phi$

If the second disk is combined (Figure 2.0) when the above value of y is placed on yz' plane, the length of z will be a half of width ($z = W_t/2$) and the apparent depth of cut can be related to the disk angle and width by:

$$H_c = r - \frac{\sqrt{4(r \sin \phi)^2 - W_t^2}}{2 \sin \phi} \dots(2)$$

Characteristics Of Furrow Slice

Area And Center Of Gravity

Figure 3.0 shows the section of furrow slice and apparent height of tillage residual that can be used to solve the problem relating to the characteristic of furrow slice.

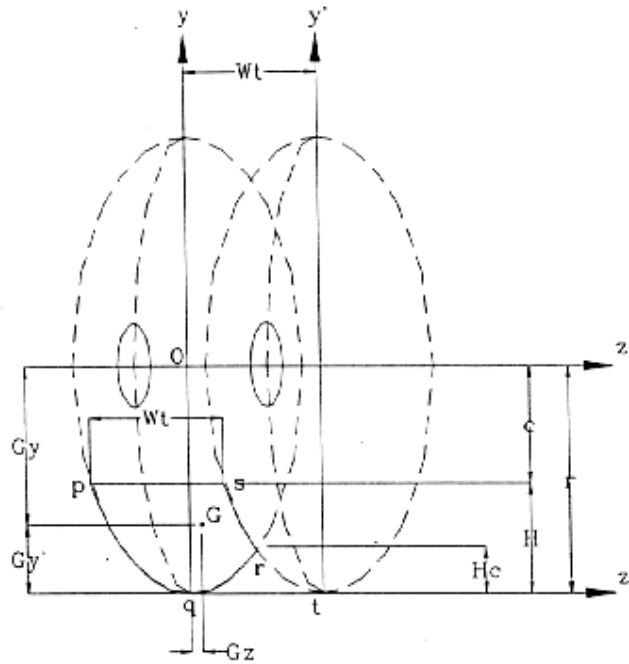


Figure 3.0 The section of furrow slice and apparent height of tillage residual

If $y = -r\cos\theta$ and $z = -r\sin\phi\sin\theta$ on yz-plane, then the length of st-plane on z-axis: $z_1 = -\sin\phi\sqrt{r^2 - y^2}$ and the total length of soil cutting on uv-plane, $z_2 = W_t + z_1$

The projected area of soil that is cut by disk on x-plane can be used to evaluate the nature and magnitude of furrow slice on the convex side of the disk. The area of furrow slice is

$$Ad = \int z_1 dy + \int z_2 dy + \frac{Hc \left\{ W_t - \sin\phi \cdot \sqrt{r^2 - (Hc - r)^2} \right\}}{2}$$

$$\text{or, } Ad = \int_{r-H}^r \sin\phi \sqrt{r^2 - y^2} dy + \int_{r-H}^{r-Hc} (W_t - \sin\phi \sqrt{r^2 - y^2}) dy + \frac{Hr \left\{ W_t \sin\phi \sqrt{r^2 - (Hc - r)^2} \right\}}{2}$$

or,

$$Ad = \frac{\pi r^2}{4} \sin\phi + W_t \left(H - \frac{Hc}{2} \right) - \frac{r \sin\phi}{2} \left[\sqrt{2rHc - Hc^2} + r \sin \left(r - \frac{Hc}{r} \right) \right] \quad \dots(3)$$

or,

$$\begin{aligned}
 I_y = & \frac{\left(Wt - \sin \phi \sqrt{2rHc - Hc^2}\right)^3 (4H - 3Hc)}{12} \dots \\
 & + 0 - \frac{\sin^2 \phi}{12} \left[\left(3 \sin \phi \sqrt{2rHc - Hc^2} - 8Wt\right)(r - H)^3 + 0 - \left(3 \sin \phi \sqrt{2rHc - Hc^2} - 8Wt\right)(r - Hc)^3 \right] \dots \\
 & + 0 - \frac{(4Wt^2 - (r \sin \phi)^2) \sin \phi}{8} \left[\left((r - H) \sqrt{2rH - H^2} - (r - Hc) \sqrt{2rHc - Hc^2} \right) \dots \right. \\
 & \left. + 0 - r^2 \left(a \sin \left(\frac{\sqrt{2rH - H^2}}{r} \right) - a \sin \left(\frac{\sqrt{2rHc - Hc^2}}{r} \right) \right) \right] \dots \\
 & + \frac{\sin^3 \phi}{24} \left[6(r - H)^3 \sqrt{2rH - H^2} - 3r^2 \left\{ (r - H) \sqrt{2rH - H^2} - r^2 a \sin \left(\frac{-\sqrt{2rH - H^2}}{r} \right) \right\} \dots \right. \\
 & \left. + 8(r - H) \sqrt{(2rH - H^2)^3} \right] \dots \dots (4)
 \end{aligned}$$

Similarly, the area moment of inertia on z-axis can be expressed as

$$\begin{aligned}
 I_z = & \int_0^{Wt - \sin \phi \sqrt{2rHc - Hc^2}} dz \int_{r + \frac{Hc \cdot z}{Wt - \sin \phi \sqrt{2rHc - Hc^2}}}^0 y^2 dy - \int_0^{Wt - \sin \phi \sqrt{2rH - H^2}} dz \int_{-(r - H)}^0 y^2 dy \dots \\
 & - \int_{Wt - \sin \phi \sqrt{2rH - H^2}}^{Wt - \sin \phi \sqrt{2rHc - Hc^2}} dz \int_{-\frac{\sqrt{(r \sin \phi)^2 - (z - Wt)^2}}{\sin \phi}}^0 y^2 dy + \int_{-\sin \phi \sqrt{2rH - H^2}}^0 dz \int_{-\frac{\sqrt{(r \sin \phi)^2 - z^2}}{\sin \phi}}^0 y^2 dy \dots \\
 & - \int_{\sin \phi \sqrt{2rH - H^2}}^0 dz \int_{-(r - H)}^0 y^2 dy \\
 \text{or,} \\
 I_z = & - \frac{\left(Wt - \sin \phi \sqrt{2rHc - Hc^2}\right)(r - Hc)^4 - r^4}{12Hc} - \frac{\left(Wt - \sin \phi \sqrt{2rH - H^2}\right)(r - H)^3}{3} \dots \\
 & + \frac{\sin \phi \sqrt{2rHc - Hc^2} (r - Hc) \{2(r - Hc)^2 + 3r^2\}}{6} \dots \\
 & + \frac{r^4 \sin \phi}{8} \left\{ a \sin \left(\frac{\sqrt{2rHc - Hc^2}}{r} \right) - a \sin \left(\frac{\sqrt{2rH - H^2}}{r} \right) \right\} \dots \\
 & - \frac{\sin \phi \sqrt{2rH - H^2} (r - H) \{2(r - H)^2 + 3r^2\}}{6} + \frac{\sin \phi \sqrt{2rH - H^2} (r - H) \{12rH - 3r^2 - 4H^2\}}{24} \dots \\
 & + \frac{3r^4 \sin \phi}{24} a \sin \left(\frac{-\sqrt{2rH - H^2}}{r} \right) \dots \dots (5)
 \end{aligned}$$

The distance apparent height to center point O is given by

$$d_{or} = \sqrt{(r - Hc^2) + (Wt - \sin \phi \sqrt{2rHc - Hc^2})} \quad \dots(6)$$

Since mass of furrow slice can be expressed as

$$m = Ad.\rho.Se \quad \dots(7)$$

and mass moment of inertia on x-axis expressed as

$$I_{xx} = \rho Se(I_y + I_z) + Ad.d_{or}^2 \quad \dots(8)$$

Therefore mass of soil can be represented by

$$m = \frac{Ad.I_{xx}}{(I_y + I_z) + Ad.d_{or}^2} \quad \dots(9)$$

and the energy requirement of a disk plow as

$$Ed = m.Vd^2 \quad \dots(10)$$

Rotary Blade

The most widely used rotary tillers are those having straight, bent and chisel shaped C-blade which provide sliding cutting of the fibrous soil with minimum resistance and have self-cleaning characteristics in working damp soils. These blades also have sufficient resistance to breakage and abrasive wear. In the process of cutting, the rotary blade hits the soil surface with sufficient cutting velocity to produce dynamic compressive and shear stress in the soil. These dynamic stresses produced under impact loading are much greater than stress produced by blade under steady loading [7]. Based on the predicted behavior of wet soil and the previous experiment by Gupta and Visvanathan [7] on rotary plow with deep tillage or reversed rotational rotary [13 and 14] and straight blade, mathematical model to predict the power requirement of a rotary blade is the sum of the energy required to cut the soil and the energy required to throw the soil as shown in Figure 5.0

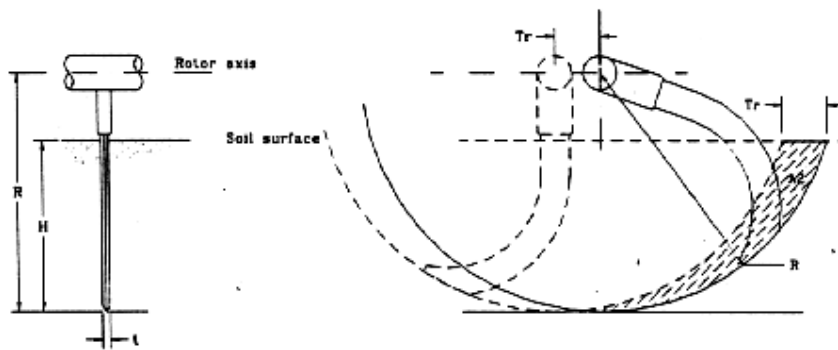


Figure 5.0 Soil cutting by rotary blade

The energy required to cut the soil can be derived from the impact shear ($\sigma = F/Ar$) where total area of blade, $Ar = H.t$. On the other hand, the energy required to throw the soil can be derived from centrifugal force ($F_c = m_s.V_r^2$) where the cutting mass of sliding soil, $m_s = A_r.T_r.\rho$.

The tilling pitch can be defined as follows:

$$Tr = \frac{Vd}{Vr} \cdot \frac{2\pi Rr}{z} \quad \dots(11)$$

Hence, the energy requirement to cut the wet soil by rotary blade can be expressed as

$$Er = \left\{ \sigma + \frac{Tr \cdot \rho}{Rr} \right\} \frac{(h.t). Vd^2}{Rr} Vr \quad \dots(12)$$

Combination Of Disk Plow And Rotary Blade

The energy requirement for combination of disk plow and rotary blade can be expressed as

$$E = Ed + Er \quad \dots(13)$$

A computer program (MathCAD PLUS 6.0) was used to solve Eqs (1 to 13) at different conditions (at various forward speeds, tilt angles, disk angles and rotary speeds). The following input parameters (Table 1) were also used to solve the equations.

Table 1: Input parameters

Parameters	Value
Soil:	
Density (ρ), kg/m ³	1650
Gravity (g), m/sec ²	9.807
Shear (σ), kg/m ²	3564
Disk Plow:	
Diameter of the disk edge circle (d), m	0.62
Rotary Blade:	
Radius of blade (R), m	0.225
Angle of blade (α), deg	45
Number of blade (z), nos	2
Thickness of blade (t), m	0.005
Operation:	
Disk spacing (Wt), m	0.2
Forward speed (Vd), m/s	0.4
Rotary speed (Vr), rpm	400
Tilt angle (β), deg	0
Depth of disk cutting (H), m	0.165
Depth of rotary cutting (h), m	0.2

$$\alpha = (\alpha/180)\pi \quad \beta = (\beta/180)\pi \quad Vr = Vr/60 \quad Se = Vd/n \quad r = d/(2\cos\beta)$$

$$c = r - H; \quad S = \sqrt{(r^2 - c^2)}; \quad \phi = a \tan(S/r); \quad \phi = (\phi/180)\pi$$

LABORATORY EXPERIMENTS

Experimental Set-Up

The study was carried out in the laboratory of the Department of Biological and Agricultural Engineering, University Putra Malaysia (UPM), Serdang. A soil tank having the track of a gantry system to carry the tractor (MITSUBISHI MT250D) and dynamometer was developed. The soil tank was 20 m long, 3 m wide and 0.75 m deep containing a tractor driven gantry system running on rail above the soil tank. A frame was designed for carrying the disk plow and rotary blade for the purpose of getting better mixing of clay soil. After setting the disk plow and rotary blade on the designed frame in the tractor gantry system, treatments of forward speeds (0.2, 0.4 and 0.6 m/s), tilt angles (0°, 10° and 20°), disk angles (40° and 50°) and widths of cut (15 and 25 cm)

were selected for the entire experiment. Similarly, rotor speeds (200 and 400 rpm) were considered to determine the position of blade on the disk shaft. The nature of soil disturbance, draft and torque requirement were investigated. It was anticipated that the results would reveal the optimum soil-disk design parameters at which tillage should be conducted to achieve the required soil disturbance under given conditions, and the most efficient design of implement be recommended. A special transducer (octagonal ring dynamometer) was made to measure the required compressive forces and moment [15-18]. The signals from octagonal ring transducer (F_x , F_z and M_y) and from the torquemeter were transferred to the computer for further analysis through a logging unit (DATATAKER 605).

The measured energy for combination of disk plow and rotary blade was calculated from the following equations:

$$E_d = F_x \times S; \quad E_r = 2 \pi \times (n/60) \times T \quad \text{and} \quad E = E_d + E_r$$

where, E_d = energy for cutting soil by disk plow, watt
 F_x = draft force, N
 S = forward speed, m/s
 E_r = energy for cutting soil by rotary blade, watt
 T = required torque to rotary blade, N-m
 n = rotational speed of rotary blade, rpm
 E = total energy, watt

Soil Type And Preparation

Muchong series clay from subsoil of 0.5 m deep was chosen in the experiment. Table 2 represents details of the physical analysis of soil. The cohesion (c) and the friction angle (ϕ) were found to be 34.9 kPa and 4.25°, respectively, during the triaxial test at 32% moisture content.

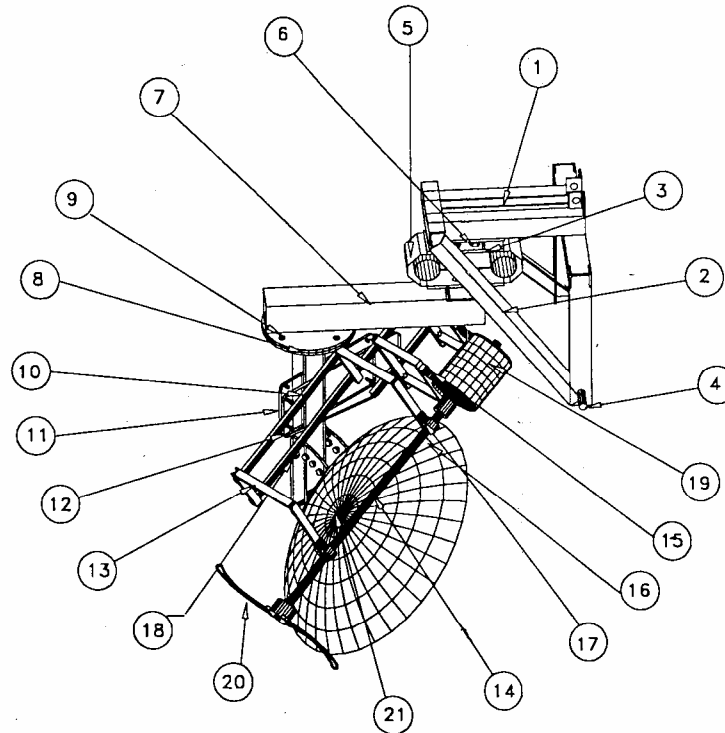
Table 2: Physical properties of Muchong series clay soil

Properties	Value (%)
Contents:	
Clay	65.69
Silt	11.34
Sand	15.08
Humus	7.88
Consistency limits:	
Liquid limit	53.65
Plastic limit	33.20
Plasticity index	20.45

The soil moisture content was chosen about 40% (dry basis) in between plastic limit and liquid limit due to suitability for puddling (Table 2). It was observed that if the moisture content was higher than 50%, the soil was very weak and wet. Before starting the test, wet clay soil from the field was initially dried and crushed into smaller aggregates. The clean soil was then poured into the tank. To prepare the soil to the required moisture content, a measured amount of water was added to a given weight of soil. The required soil moisture content was maintained by flooding, harrowing, and levelling operations and then the soil was left for 3 days to reach around 40% moisture content. During the tests, the moisture content of the soil in the tank was monitored every day and water was added to maintain the required homogeneity. The experimental soil was prepared in the same way for every test.

Disk-Rotary Blade Implement

The disk implement consisted of a standard disk of 61 cm diameter and its frame was designed in such a way that the disk angle and tilt angle could be varied (Figure 6.0). The depth of tillage was set to about a quarter of the disk diameter (15 cm).



All dimensions are in mm

1 - Sub Frame 1 (U-Shape 75x50)	8 - Sub Frame 2-3 (Plate 10)	16 - Pillow Block (dia.25)
2 - Sub Frame 1 (Angle L 75x50)	9 - Bolt & nut	17 - Bolt & Nut
3 - Sub Frame 1 (Plate 10)	10 - Sub Frame 3 (U-shape 125x60)	18 - Bolt & Nut
4 - Sub Frame 1 (Solid dia.25)	11 - Sub Frame 3-4 (Plate 10)	19 - Torquemeter
5 - Dynamometer (150x150x400)	12 - Bolt & Nut	20 - Blade and Base (225x5xR125)
6 - Bolt	13 - Sub Frame 4 (U-shape 50x25)	21 - Disk and Bearing (dia.600)
7 - Sub Frame 2 (Sq.-Shape 100x100)	14 - Sub Frame 4 (Solid dia.25)	
	15 - Bolt & Nut	

Figure 6.0 Unit frame containing disk plow, rotor blade and transducers in tractor hitching frame

Disk Plow-Rotary Blade Implement Interaction

The position of C-shaped rotary blade was determined according to the upward soil movement at the end of the disk. The rotation of blade was reversed which had an advantage to cut soil deeper and reduced the pressure on disk [13]. The axle of rotary blade was fixed at 45 deg towards the movement of soil cutting on disk surface and it was driven by an electric variable speed motor (1.47 kW). The rotational speeds were selected at 0 rpm, 200 rpm and 400 rpm.

RESULTS AND DISCUSSION

Using the statistical method, the analysis of variance was done to check the significance of interrelationship among obtained data sets. It was found from the experiment that the force and energy depended on forward speed and width of cut. A reverse effect was found when the rotor speed increased. The effect of tilt angle on the energy was almost none probably due to the wet soil. However, calculations were done on the total energy both from the theoretical model and the experimental values in respect of various forward speeds, rotor speeds,

disk angles and tilt angles. Discussion is only given on the variation of energy from predicted and measured forms in respect of rotor speeds for 200 and 400 rpm due to limitation of space to present the huge data.

Energy Variation At Rotor Speed Of 200 Rpm

Figure 7.0 represents the variation of energy requirement at different forward speeds with 40 deg disk angle and 0 deg tilt angle at 25 cm width of cut with a rotor speed of 200 rpm. The total energy increased with the increase of forward speed from 0.2m/s to 0.6m/s. The total energy in measured and predicted forms also increased with increasing forward speed. The variation ranged from -0.6% to +3.3%. The measured values were found to be higher at forward speeds of 0.4 and 0.6 m/s. It could be assumed that at a lower forward speed (0.2 m/s), the initial energy requirement was comparatively lower than those at higher forward speeds and the measured energy was slightly lower or almost equal to the predicted value. However, when the forward speed was increased, the energy requirement increased and due to the effects of other uncontrolled factors, the measured energy was higher than the predicted one.

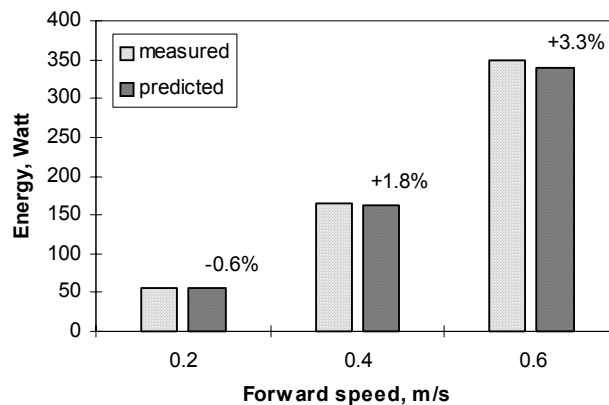


Figure 7.0 Energy variation at a rotor speed of 200 rpm

Energy Variation At Rotor Speed Of 400 Rpm

Similar trend was observed in the energy variation at different forward speeds with 40 deg disk angle and 0 deg tilt angle at 25 cm of width of cut with a rotor speed of 400 rpm (Figure 8.0). At this rotor speed, the variation was higher compared to the previous one (Figure 7.0) and it varied from -1.4% to +2.5%. Since, rotary tiller always exert a negative draft on the prime mover while in operation [8], the energy requirement reduced when the rotary speed was increased from 200 to 400 rpm (Figures 7.0 & 8.0). Similarly, the increase of total energy was not much higher at 0.2 and 0.4 m/s compared to 0.6 m/s forward speed. Consequently the variation of energy in measured and predicted forms was not so different from the case of rotor speed at 200 rpm. However, it was evident that the increase of rotor speeds obviously increased the specific torque and at the same time, reduced the draft force.

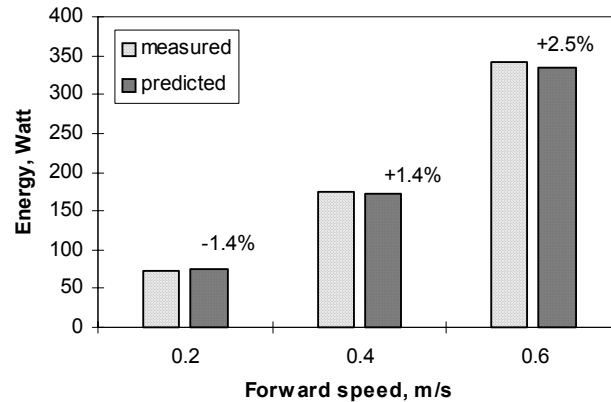


Figure 8.0 Energy variation at a rotor speed of 400 rpm

It was noted that rotor speed helped reduce the combined draft because of its forward thrust on the machine. It was also observed that the soil clogging increased with decrease of forward speed. On the other hand, if the disk angle was increased from 40 deg to 50 deg the disk penetration and width of cut increased and at the same time increased its energy requirement. Tilt angle variation had almost no effect on the energy requirement. Increasing tilt angle moved more soil upward and forward which made better soil overturning. Generally, the increase of tilt angle reduces penetration in dry soil, however, in wet soil the situation is reversed.

CONCLUSION

Experiments were conducted in Muchong clay series soil of Selangor, Malaysia at 40% moisture content using combined disk plow and C-shaped rotary blade. Based on the theoretical analysis and experimental observation, a mathematical model for predicting the energy requirement was developed. It was found that the variation between measured energy and predicted energy varied from -1.4% to 3.3% which is not significant and thereby, the model can be used to improve the machine capabilities for estimating energy requirement in soil manipulation.

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List of Symbols

β	tilt angle of disk	M_y	moment on y-axis
ϕ	disk angle	m	mass of furrow slice
θ	angle between soil surface and horizontal	n	rotation of disk
ρ	soil density	R	radius of blade
σ	soil shear	r	radius of the disk edge circle
A_d	area of disk furrow slice	S	horizontal distance between soil surface and horizontal
A_r	area of rotary blade	Se	soil exposure
c	vertical distance of soil surface	Tr	tilling pitch
d	diameter of disk	t	thickness of blade
d_{or}	apparent height of soil slice to center point	V_d	forward speeds
F_x, F_z	force	V_r	rotary speed
H	tilling depth	W_t, W_d	cutting width
H_c	apparent height of cut	z	number of blade
I_x, I_y, I_z	area of moment of inertia		
I_{xx}, I_{yy}, I_{zz}	mass moment of inertia		